

Solução da 2ª Chamada

$$1. \quad (a) \quad \lim_{x \rightarrow 3^-} \frac{x^2 - 4x + 4}{x^2 - 5x + 6} = \frac{0}{0} = -\infty \quad (0.4)$$

$$(b) \quad \lim_{x \rightarrow 3^-} \frac{x^2 - 4x + 3}{x^2 - 5x + 6} = \frac{0}{0} = \lim_{x \rightarrow 3^-} \frac{(x-3)(x-1)}{(x-3)(x-2)} = \lim_{x \rightarrow 3^-} \frac{x-1}{x-2} = \frac{2}{1} = 2 \quad (0.5)$$

$$(c) \quad \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2} \quad (0.6)$$

$$(1.5)$$

$$2. \quad (a) \quad f(x) = 2x^8 - 5x^3 + 6x + 7, \quad f'(x) = 16x^7 - 15x^2 + 6 + 0 \quad (0.4)$$

$$(b) \quad g(y) = \cos(e^{20y}), \quad g'(y) = -\operatorname{sen}(e^{20y}) \cdot e^{20y} \cdot 20 \quad (0.4)$$

$$(c) \quad h(t) = \frac{\operatorname{senh} t}{1 + \cosh t}, \quad h'(t) = \frac{\cosh t(1 + \cosh t) - \operatorname{senh} t \operatorname{senh} t}{(1 + \cosh t)^2} =$$

$$\frac{\cosh t + \cosh^2 t - \operatorname{senh}^2 t}{(1 + \cosh t)^2} = \frac{\cosh t + 1}{(1 + \cosh t)^2} = \frac{1}{1 + \cosh t} \quad (0.6)$$

$$(d) \quad k(z) = \operatorname{arcsen} z \cdot \ln(8y) \cdot 2^z, \quad k'(z) = \frac{1}{\sqrt{1-z^2}} \cdot \ln(8y) \cdot 2^z$$

$$+ \operatorname{arcsen} z \cdot \frac{1}{8z} \cdot 8 \cdot 2^z + \operatorname{arcsen} z \cdot \ln(8y) \cdot 2^z \ln 2 \quad (0.6)$$

$$(2.0)$$

$$3. \quad (a) \quad I_a = \int x^3 \sqrt{x^2 + 1} dx : \quad u = x^2 + 1, \quad du = 2x dx, \quad x dx = du/2, \quad x^2 = u - 1$$

$$I_a = \frac{1}{2} \int (u-1) \sqrt{u} du = \frac{1}{2} \int u^{3/2} du - \frac{1}{2} \int u^{1/2} du = \frac{1}{2} \frac{u^{5/2}}{5/2} - \frac{1}{2} \frac{u^{3/2}}{3/2} + C =$$

$$\frac{1}{5} (x^2 + 1)^{5/2} - \frac{1}{3} (x^2 + 1)^{3/2} + C = \frac{1}{15} \sqrt{x^2 + 1} (3x^4 + x^2 - 2) + C \quad (0.9)$$

$$(b) \quad I_a = \frac{1}{x^2 - 2x - 8} dx = \int \frac{1}{(x+2)(x-4)} dx.$$

Frações parciais: $\frac{1}{(x+2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-4} = \frac{A(x-4) + B(x+2)}{(x-4)(x+2)} =$

$$\frac{(A+B)x - 4A + 2B}{(x-4)(x+2)}. \text{ Assim, } A+B=0 \Rightarrow B=-A \text{ e } -4A+2B=1 \Rightarrow$$

$$-4A-2A=1 \Rightarrow A=-1/6 \Rightarrow B=1/6.$$

$$I_b = -\frac{1}{6} \int \frac{1}{x+2} dx + \frac{1}{6} \int \frac{1}{x-4} dx = -\frac{1}{6} \ln|x+2| + \frac{1}{6} \ln|x-4| + C =$$

$$\frac{1}{6} \ln C \left| \frac{x-4}{x+2} \right| \quad (0.8)$$

$$(c) \quad I_c = \frac{1}{x^2 + 6x + 10} dx = \int \frac{1}{(x+3)^2 + 1} dx : \quad u = x+3, \quad du = dx$$

$$I_c = \int \frac{1}{u^2 + 1} du = \arctan u + C = \arctan(x+3) + C \quad (0.7)$$

$$(2.4)$$

$$4. \quad \text{Numerador: } I_N = \int_0^1 x \operatorname{sen} \pi x dx. \text{ Intergração por partes: } f(x) = x, \quad f'(x) = 1,$$

$$g'(x) = \operatorname{sen} \pi x, \quad g(x) = -\frac{1}{\pi} \cos \pi x. \text{ Assim, } I_N = -\frac{x}{\pi} \cos \pi x \Big|_0^1 + \frac{1}{\pi} \int_0^1 \cos \pi x dx =$$

$$-\frac{1}{\pi}(1 \cdot (-1) - 0 \cdot 1) + \frac{1}{\pi^2} \operatorname{sen} \pi x \Big|_0^1 \checkmark = \frac{1}{\pi} + \frac{1}{\pi^2}(0 - 0) = \frac{1}{\pi} \checkmark \quad (0.6)$$

$$\text{Denominador: } I_D = \int_0^1 \operatorname{sen} \pi x \, dx \checkmark = -\frac{1}{\pi} \cos \pi x \Big|_0^1 \checkmark = -\frac{1}{\pi}(-1 - 1) \checkmark = \frac{2}{\pi} \checkmark \quad (0.5)$$

$$\text{Portanto, } C_M = \frac{1/\pi}{2/\pi} = \frac{1}{2} \checkmark \quad (0.1)$$

5. (a) Domínio: $y(0) \nexists \Rightarrow \mathbb{D} = \mathbb{R} - 0 \checkmark \quad (0.1)$

(b) $0 \notin \mathbb{D} \Rightarrow$ não tem intercepto y .

$$y(x) = x + 5 + \frac{4}{x} = \frac{(x+4)(x+1)}{x} = 0 \text{ em } x = -1 \checkmark \text{ e } x = -4 \checkmark \quad (0.2)$$

(c) Pontos críticos: $y'(x) = 1 - \frac{4}{x^2} \checkmark = \frac{x^2 - 4}{x^2} = \frac{(x+2)(x-2)}{x^2} = 0 \checkmark$ em $x = \pm 2 \checkmark$

$y'(x) \nexists$ em $x = 0$, mas este ponto não faz parte do domínio da função. \checkmark

$$y''(x) = \frac{8}{x^3} \checkmark \Rightarrow y''(-2) = \frac{8}{-8} = -1 \checkmark < 0 \Rightarrow \text{máximo relativo} \checkmark.$$

$$y''(x) = \frac{8}{x^3} \Rightarrow y''(2) = \frac{8}{8} = 1 \checkmark > 0 \Rightarrow \text{mínimo relativo} \checkmark. \quad (0.9)$$

(d) $y''(x) \neq 0 \checkmark \forall x \in \mathbb{D} \Rightarrow$ não há pontos de inflexão $\checkmark. \quad (0.2)$

(e) Assíntotas verticais:

$$\lim_{x \rightarrow 0^+} x + 5 + \frac{4}{x} \checkmark = 5 + \frac{4}{0^+} = +\infty \checkmark \text{ e } \lim_{x \rightarrow 0^-} x + 5 + \frac{4}{x} \checkmark = 5 + \frac{4}{0^-} = -\infty \checkmark$$

Portanto, a reta $x = 0$ é assíntota vertical \checkmark .

$$\text{Assíntotas horizontais: } \lim_{x \rightarrow \infty} x + 5 + \frac{4}{x} = \infty \text{ e } \lim_{x \rightarrow -\infty} x + 5 + \frac{4}{x} = -\infty$$

Portanto, a função não há assíntotas horizontais.

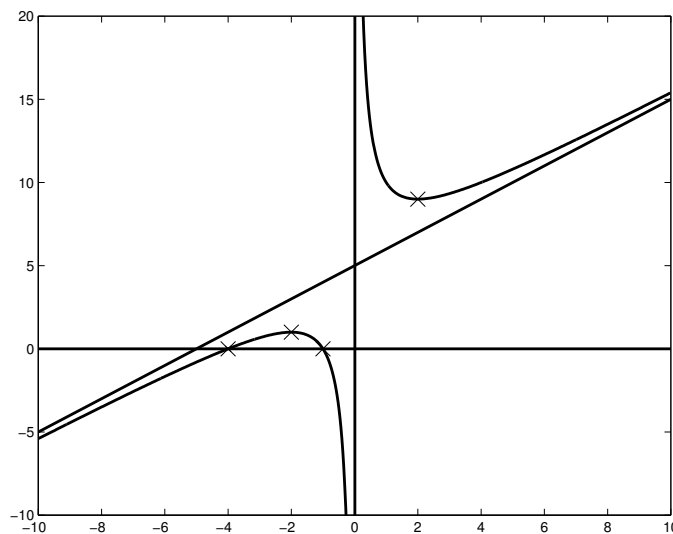
$$\text{Assíntotas inclinadas: } \lim_{x \rightarrow \pm\infty} 1 - \frac{4}{x^2} \checkmark = 1 \checkmark = m$$

$$\lim_{x \rightarrow \pm\infty} x + 5 + \frac{4}{x} - mx \checkmark = \lim_{x \rightarrow \pm\infty} x + 5 + \frac{4}{x} - x = \lim_{x \rightarrow \pm\infty} 5 + \frac{4}{x} = 5 \checkmark = b$$

Portanto, a reta $y = x + 5$ é assíntota inclinada para $x \rightarrow \pm\infty \checkmark. \quad (1.0)$

(f) Imagem: $\mathbb{V} = \mathbb{R} - (1, 9) \checkmark \quad (0.1)$

(g) Gráfico:



Zeros \checkmark
 Extremos \checkmark
 Comportamento no polo \checkmark
 Comportamento assintótico \checkmark

$$(0.4)$$

$$(2.9)$$

$$(10.0)$$