AO $xy' + 6y = 3xy^{4/3}$

$$y' + \frac{6}{x} y = 3y^{4/3}$$

2

$$\underbrace{y^{-4/3} y'}_{\downarrow} + \frac{6}{x} \underbrace{y y^{-4/3}}_{\to y^{-1/3}} = 3$$

$y^{-1/3} = v$

$\underbrace{-\frac{1}{3} y^{-4/3} y'}= v'$

$$-3v' + \frac{6}{x} v = 3$$

2

$$v' - \frac{2}{x} v = -1$$

$$\mu(x) = e^{\int -\frac{2}{x} dx} = e^{-2 \ln x} = x^{-2}$$

2

$$x^{-2} v' - x^{-2} \frac{2}{x} v = -x^{-2}$$

$$\frac{d}{dx}\left(x^{-2} v\right) = -x^{-2}$$

$\xrightarrow{\text{integrar } \int \cdot\, dx}$

$$x^{-2} v = +x^{-1} + C$$

2

$$v = x + Cx^2$$

$$\boxed{y^{-1/3} = x + Cx^2}$$

2

A1

$$\underbrace{2x}_{M}\,dx + \underbrace{x^2 \cot g\, y}_{N}\,dy = 0$$

2

$M_y = 0 \neq N_x = 2x \cot g\, y$ não é exata

$$\frac{M_y - N_x}{N} = \frac{0 - 2x \cot g\, y}{x^2 \cot g\, y} = -\frac{2}{x}$$

2

fator integrante $e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = x^{-2}$

$$2x\,x^{-2}\,dx + \cot g\, y\, dy = 0$$

2

$M_y = 0 = N_x$ a edo é exata!

$\Rightarrow$ existe $F(x,y) = C$ tal que $F_x = M$ $F_y = N$

$$F = \int M\,dx = \int 2x^{-1}\,dx = 2\ln x + g(y)$$

$$F_y = g'(y) = \cot g\, y$$

2

$$g(y) = \ln|\operatorname{sen} y|$$

A solução é $F(x,y) = C$

$$\boxed{2\ln x + \ln|\operatorname{sen} y| = C}$$

2

A2 $\quad yy'' + (y')^2 = yy'$

$v(y) = y' \Rightarrow y'' = v'(y)\,y' =$ $\boxed{2}$

$= v'v = y''$

$y v' v + v^2 = y v$

$v' + \dfrac{v^2}{y \cdot v} = 1$ $\boxed{2}$

$v' + \dfrac{1}{y} v = 1 \quad \leftarrow$ linear em $v(y)$.

fat. int:

$\mu(y) = e^{\int \frac{1}{y} dy} = e^{\ln y} = y.$

$y v' + v = y$

$\dfrac{d}{dy}[v y] = y$ integrar $\int \cdot \, dy$ $\boxed{2}$

$v y = \dfrac{y^2}{2} + C \Rightarrow v = \dfrac{y}{2} + C y^{-1}$

$\Rightarrow y' = \dfrac{y^2 + C}{2y} \xrightarrow{\text{separável}} \displaystyle\int \frac{2y}{y^2 + C}\, dy = \int dx$ $\boxed{2}$

$\boxed{\ln |y^2 + C| = x + D} \leftarrow$ $\boxed{2}$

BO $y^{(4)} - 4y^{(2)} = xe^x + x^2$

Homogênea associada $y^{(4)} - 4y^{(2)} = 0$ $\quad$ $y = e^{rx}$ candidata à solução

Eq. característica $r^4 - 4r^2 = r^2(r^2-4) =$

$= r^2(r-2)(r+2) = 0$ $\boxed{2}$ $\quad$ $\boxed{2}$

$r=0$ com multiplicidade 2 $\Rightarrow (c_1 + c_2 x)e^{0x}$

↑ multiplica-se por um polinômio de grau um a menos que a multiplicidade da raiz $r=0$.

$$\boxed{y_c = (c_1 + c_2 x) + c_3 e^{2x} + c_4 e^{-2x}}$$

Método dos coeficientes indeterminados

$$y_p = x^s (A + Bx)e^x + x^l(C + Dx + Ex^2)$$

$s=0$ $\quad$ $l=2$ ← essas escolhas eliminam repetições de termos em y_c. $\boxed{2}$

$$y_p = (A+Bx)e^x + Cx^2 + Dx^3 + Ex^4$$
$$y_p' = Be^x + (A+Bx)e^x + 2Cx + 3Dx^2 + 4Ex^3$$
$$y_p'' = 2Be^x + (A+Bx)e^x + 2C + 6Dx + 12Ex^2$$
$$y_p''' = 3Be^x + (A+Bx)e^x + 6D + 24Ex$$
$$y_p^{(4)} = 4Be^x + (A+Bx)e^x + 24E$$

$\boxed{2}$

Substituir na edo

$$y_p^{(4)} - 4y_p^{(2)} = \left[4Be^x + (A+Bx)e^x + 24E\right] - 4\left[2Be^x + (A+Bx)e^x + 2C + 6Dx + 12Ex^2\right] = xe^x + x^2$$

$$\Rightarrow (4B + A - 8B - 4A)e^x + (B - 4B)xe^x + 24E - 8C - 24Dx - 48Ex^2 = xe^x + x^2$$

$-4B - 3A = 0$

$-3B = 1$

$\boxed{B = -\frac{1}{3} \quad A = \frac{4}{9}}$

$24E - 8C = 0 \Rightarrow -\frac{1}{2} = 8C$

$-24D = 0$

$-48E = 1$

$\boxed{D = 0 \quad E = -\frac{1}{48}}$

$\boxed{C = -\frac{1}{16}}$

$\boxed{2}$

$$\boxed{y_p = \left(\frac{4}{9} - \frac{1}{3}x\right)e^x - \frac{1}{16}x^2 - \frac{1}{48}x^4}$$

B1 $\quad y''' - y' = x$

homogenea associada $y''' - y' = 0$ candidata à solução e^{rx},

$r^3 - r = r(r^2-1) = 0 \quad r = 0 \quad r = \pm 1$ $\quad \boxed{2}$

$$y_c = c_1 \underbrace{e^{0x}}_{1} + c_2 e^x + c_3 e^{-x}$$

$$y_p = u_1 + u_2 e^x + u_3 e^{-x}$$

o sistema
$$\begin{cases} u_1' + u_2' e^x + u_3' e^{-x} = 0 \\ 0 + u_2' e^x - u_3' e^{-x} = 0 \\ 0 + u_2 e^x + u_3' e^{-x} = x \end{cases} \qquad \boxed{2}$$

$$W = \begin{vmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x} \end{vmatrix} = e^x \cdot e^{-x} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 2$$

Crammer: $\qquad \boxed{2} + \boxed{2}$

$$u_1' = \frac{\begin{vmatrix} 0 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ x & e^x & e^x \end{vmatrix}}{2} = \frac{x e^x e^{-x}}{2} \begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \frac{-2x}{2} = -x$$

$$u_2' = \frac{\begin{vmatrix} 1 & 0 & e^{-x} \\ 0 & 0 & -e^{-x} \\ 0 & x & e^x \end{vmatrix}}{2} = \frac{x e^{-x}}{2} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{vmatrix} = \frac{x e^{-x}}{2}$$

$$u_3' = \frac{\begin{vmatrix} 1 & e^x & 0 \\ 0 & e^x & 0 \\ 0 & e^x & x \end{vmatrix}}{2} = \frac{x e^x}{2} \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \frac{x e^x}{2}.$$

$u_1 = \int -x = -\frac{x^2}{2}$ $\qquad u_2 = \frac{1}{2}\int x e^{-x} dx = \frac{1}{2}\left(-x e^{-x} + \int e^{-x} dx\right) = -\frac{1}{2}\left(x e^{-x} + e^{-x}\right)$

$u = x \quad du = dx$

$dv = e^{-x} dx \quad v = -e^{-x}$

$\boxed{2}$

$u_3 = \frac{1}{2}\int x e^x dx$ $\quad = \frac{1}{2}\left(x e^x - e^x\right)$

$u = x \quad dv = e^x$

$du = dx \quad v = e^x$

$$y_p = -\frac{x^2}{2} - \frac{1}{2}\left(x e^{-x} + e^{-x}\right) \cdot e^x + \frac{1}{2}\left(x e^x - e^x\right) \cdot e^{-x}$$

$$= -\frac{x^2}{2} - \frac{1}{2}(x+1) + \frac{1}{2}(x-1) = -\left(\frac{x^2}{2} + 1\right)$$

B2 $(1-x)y'' + xy' - y = -(x-1)^2 e^x \cos x$ $\quad y_1 = e^x$ e $y_2 = x$ são soluções da homogênea associada. Substituindo:

$$(1-x)y_1'' + xy_1' - y_1 = (1-x)e^x + xe^x - e^x = 0$$
$$(1-x)y_2'' + xy_2' - y_2 = (1-x).0 + x.1 - x = 0$$
$$y_c = c_1 e^x + c_2 x$$

$\boxed{2}$

Variação de parametros $\quad y_p = u_1 e^x + u_2 x$

sistema:
$$\begin{cases} u_1' e^x + u_2' x = 0 \\ u_1' e^x + u_2' = -\dfrac{(x-1)^2 e^x \cos x}{-(x-1)} = +(x-1)e^x \cos x \end{cases}$$

$$W = \begin{vmatrix} e^x & x \\ e^x & 1 \end{vmatrix} = e^x - xe^x = e^x(1-x)$$

$\boxed{2}$

$$u_1' = \frac{\begin{vmatrix} 0 & x \\ (x-1)e^x\cos x & 1 \end{vmatrix}}{e^x(1-x)} = -\frac{x(x-1)e^x\cos x}{e^x(1-x)} = x\cos x$$

$\boxed{2}$

$$u_2' = \frac{\begin{vmatrix} e^x & 0 \\ e^x & (x-1)e^x\cos x \end{vmatrix}}{e^x(1-x)} = \frac{e^{2x}(x-1)\cos x}{e^x(1-x)} = -e^x\cos x$$

$\boxed{2}$

$$u_1 = \int \underbrace{x}_{u} \underbrace{\cos x\, dx}_{dv} = x\,\text{sen}\,x + \cos x$$

$-\int \text{sen}\,x\, dx$

$du = dx \quad v = \text{sen}\,x$

$\boxed{2}$

$$u_2 = \int -\underbrace{e^x}_{u} \underbrace{\cos x\, dx}_{dv} = -\left[e^x\,\text{sen}\,x - \int (\text{sen}\,x)\underbrace{e^x}_{u} dx \right]$$

$du = e^x dx \quad v = \text{sen}\,x$

$u = e^x \quad dv = \text{sen}\,x\,dx$

$du = e^x dx \quad v = -\cos x$

$$= -\left[e^x\,\text{sen}\,x - \left(-\cos x . e^x + \int \cos x . e^x dx \right) \right]$$

Chamando $\int e^x \cos x\, dx = I$

$$\Rightarrow -I = -e^x\,\text{sen}\,x - e^x\cos x + I \Rightarrow -2I = -(e^x\,\text{sen}\,x + e^x\cos x)$$

$$I = \left(\frac{e^x\,\text{sen}\,x + e^x\cos x}{2} \right) \Rightarrow y_p = (x\,\text{sen}\,x + \cos x)e^x - \frac{xe^x}{2}(\text{sen}\,x + \cos x)$$

$$\boxed{y_p = \frac{xe^x\,\text{sen}\,x}{2} + e^x\cos x - \frac{xe^x}{2}\cos x}$$

CO $\quad y'' - y = 20\,\delta(t-2) + 6t \quad y(0) = 3 \quad y'(0) = 7$

$$s^2 Y(s) - s\,y(0) - y'(0) - Y(s) = 20\,e^{-2s} + \frac{6}{s^2}$$

$$(s^2-1)\,Y(s) = 20e^{-2s} + \frac{6}{s^2} + 3s + 7$$

$$Y(s) = \overbrace{\frac{20e^{-2s}}{s^2-1}}^{a} + \overbrace{\frac{6}{s^2(s^2-1)}}^{b} + \overbrace{\frac{3s+7}{s^2-1}}^{c} \qquad \boxed{2}$$

(a) $\mathcal{L}^{-1}\left\{\frac{1}{s^2-1}\right\} = \operatorname{senh} t$ (tabela)

$20\,\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2-1}\right\} = 20\,u_2(t)\,\operatorname{senh}(t-2)$ Usando: $u_c(t)\,f(t-c) = \mathcal{L}^{-1}\{e^{-cs}F(s)\}$

$\boxed{2}$

(b) $\mathcal{L}^{-1}\left\{\frac{6}{s^2(s^2-1)}\right\} = 6\,(\operatorname{senh} t - t)$ Usando: $\mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(\tau)\,d\tau$

$$\mathcal{L}^{-1}\left\{\frac{\frac{1}{s^2-1}}{s}\right\} = \int_0^t \operatorname{senh}\tau\,d\tau = \cosh\tau\Big|_0^t = \cosh t - 1$$

$$\mathcal{L}^{-1}\left\{\frac{\frac{1}{s(s^2-1)}}{s}\right\} = \int_0^t \cosh\tau - 1\,d\tau = \operatorname{senh}\tau - \tau\Big|_0^t = \operatorname{senh} t - t \qquad \boxed{2}$$

(c) $3\,\mathcal{L}^{-1}\left\{\frac{s}{s^2-1}\right\} + 7\,\mathcal{L}^{-1}\left\{\frac{1}{s^2-1}\right\} = 3\cosh t + 7\operatorname{senh} t$ $\boxed{2}$

$$y(t) = \mathcal{L}^{-1}(Y(s))$$

$$y(t) = 20\,u_2(t)\,\operatorname{senh}(t-2) + 6\,(\operatorname{senh} t - t) + 3\cosh t + 7\operatorname{senh} t$$

$\boxed{2}$

C1 $\quad 2y'' + 4y' + 10y = (\cos t)\,\delta_2(t) \quad y(0) = 4 \quad y'(0) = 5$

$$2\big(s^2 Y(s) - s\underbrace{y(0)}_{4} - \underbrace{y'(0)}_{5}\big) + 4\big(sY(s) - \underbrace{y(0)}_{4}\big) + 10Y(s) = \mathcal{L}\{\delta_2(t)\cos t\}$$

$$\mathcal{L}\{\cos t \cdot \delta_2(t)\} = \int_0^\infty (e^{-st}\cos t)\,\delta_2(t)\,dt$$

definição de transformada de Laplace

$$= e^{-2s}\cos 2$$

definição de Delta de Dirac

$\boxed{2}$

$$(2s^2 + 4s + 10)\,Y(s) = (\cos 2)\,e^{-2s} + 8s + 26$$

$$Y(s) = \frac{(\cos 2)e^{-2s}}{2(s^2+2s+5)} + \frac{8s+26}{2(s^2+2s+5)}$$

$$= \underbrace{\frac{\cos 2}{2}\,\frac{e^{-2s}}{(s+1)^2+4}}_{a} + \underbrace{\frac{4s+13}{(s+1)^2+4}}_{b}$$

$\boxed{2}$

(a) $\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+4}\right\} = \frac{1}{2}e^{-t}\,\mathrm{sen}\,2t$ (tabela) $\boxed{2}$

$$\frac{\cos 2}{2}\mathcal{L}^{-1}\left\{e^{-2s}\frac{1}{(s+1)^2+4}\right\} = \frac{\cos 2}{2}\,u_2(t)\,\frac{1}{2}e^{-(t-2)}\,\mathrm{sen}\,2(t-2)$$

usando $\mathcal{L}^{-1}\{e^{-cs}F(s)\} = u_c(t)\,f(t-c)$ p/ $c = 2$

(b) $\mathcal{L}^{-1}\left\{\frac{4s+13}{(s+1)^2+4}\right\} = 4\mathcal{L}^{-1}\left\{\frac{s+1+9/4}{(s+1)^2+4}\right\} =$ $\boxed{2}$

$$4\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\} + \frac{9}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+4}\right\} = 4e^{-t}\cos 2t + \frac{9}{2}e^{-t}\,\mathrm{sen}\,2t$$

tabela

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{\cos 2}{4}\,u_2(t)\,e^{-(t-2)}\,\mathrm{sen}\,2(t-2) + 4e^{-t}\cos 2t + \frac{9}{2}e^{-t}\,\mathrm{sen}\,2t$$

$\boxed{2}$

C2 $y'' + 4y' + 13y = 1 + \delta_8(t) \quad y(0) = 3 \quad y'(0) = 2$

$\mathcal{L}$

$$s^2 Y(s) - s\underbrace{y(0)}_{3} - \underbrace{y'(0)}_{2} + 4\left(sY(s) - \underbrace{y(0)}_{3}\right) + 13Y(s) = \frac{1}{s} + e^{-8s}$$

$$\underbrace{(s^2 + 4s + 13)}_{(s+2)^2+9} Y(s) = \frac{1}{s} + e^{-8s} + 3s + 14$$

$\boxed{2}$

$$Y(s) = \underbrace{\frac{1}{s[(s+2)^2+9]}}_{a} + \underbrace{\frac{e^{-8s}}{(s+2)^2+9}}_{b} + \underbrace{\frac{3s+14}{(s+2)^2+9}}_{c}$$

(a) $\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2+9}\right\} = \frac{1}{3}e^{-2t}\operatorname{sen}3t$ tabela.

$$\frac{1}{s[(s+2)^2+9]} = \frac{A}{s} + \frac{Bs+C}{(s+2)^2+9} = \frac{A(s^2+4s+13) + Bs^2 + Cs}{s[(s+2)^2+9]}$$

$$\left.\begin{array}{l} A+B=0 \\ 4A+C=0 \\ 13A=1 \end{array}\right\} \Rightarrow A = \frac{1}{13} \Rightarrow B = -\frac{1}{13} \quad C = -\frac{4}{13}.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s[(s+2)^2+9]}\right\} = \mathcal{L}^{-1}\left\{\frac{1/13}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{\frac{1}{13}s + \frac{4}{13}}{(s+2)^2+9}\right\} =$$

$$\frac{1}{13} + \frac{1}{13}\mathcal{L}^{-1}\left\{\frac{(s+2)+2}{(s+2)^2+9}\right\} = \frac{1}{13}\left(1 + e^{-2t}\cos 3t + \right.$$

$\boxed{2}$

$$\left. + \frac{2}{3}e^{-2t}\operatorname{sen}3t\right)$$

(b) $\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{e^{-8s} \cdot 3}{(s+2)^2+9}\right\} = \frac{1}{3}u_8(t)f(t-8) =$

$$= \frac{1}{3}u_8(t)e^{-2(t-8)}\operatorname{sen}3(t-8)$$

$\boxed{2}$

(c) $\mathcal{L}^{-1}\left\{\frac{3(s+2)+8}{(s+2)^2+9}\right\} = 3\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+9}\right\} + \frac{8}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2+9}\right\}$

$$= 3e^{-2t}\cos 3t + \frac{8}{3}e^{-2t}\operatorname{sen}3t$$

$\boxed{2}$

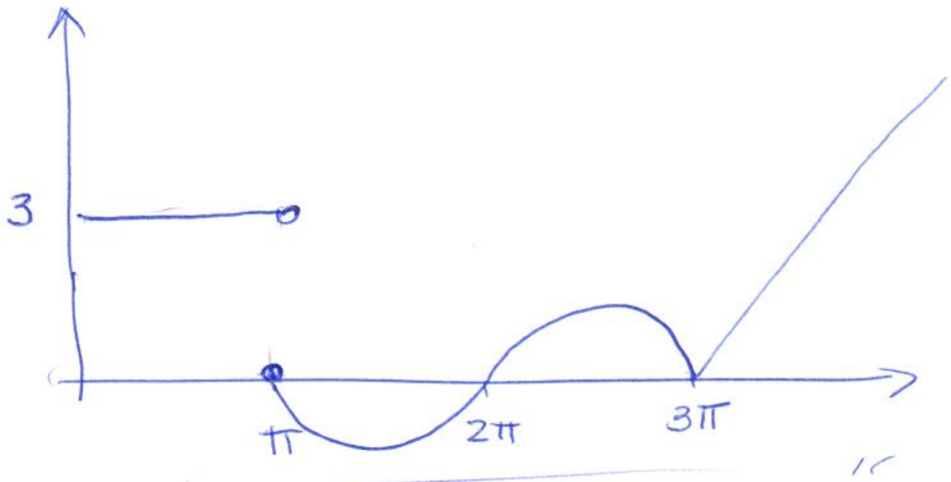
$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = \frac{1}{13}\left(1 + e^{-2t}\cos 3t + \frac{2}{3}e^{-2t}\operatorname{sen}3t\right) +$$

$$+ \frac{1}{3}u_8(t)e^{-2(t-8)}\operatorname{sen}3(t-8) + 3e^{-2t}\cos 3t + \frac{8}{3}e^{-2t}\operatorname{sen}3t$$

$\boxed{2}$

DO

$$f(t) = \begin{cases} 3 & 0 \leq t < \pi \\ \operatorname{sen} t & \pi \leq t < 3\pi \\ t - 3\pi & t \geq 3\pi \end{cases}$$



(fora de escala)

$\boxed{2}$

$$f(t) = 3(1 - u_\pi(t)) + u_\pi(t) \operatorname{sen} t - u_{3\pi}(t) \operatorname{sen} t + u_{3\pi}(t)(t - 3\pi)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3 - 3u_\pi(t)\} + \mathcal{L}\{u_\pi(t) \operatorname{sen} t\} - \mathcal{L}\{u_{3\pi}(t) \operatorname{sen} t\} + \mathcal{L}\{u_{3\pi}(t)(t - 3\pi)\}$$ $\boxed{2}$

$$\mathcal{L}\{3 - 3u_\pi(t)\} = \frac{3}{s} - \frac{3e^{-\pi s}}{s}$$ $\boxed{2}$

$$\mathcal{L}\{u_\pi(t) \underbrace{\operatorname{sen} t}_{f(t-\pi)}\} = e^{-\pi s} \mathcal{L}\{-\operatorname{sen} t\} = -e^{-\pi s} \frac{1}{s^2+1}$$

$f(t-\pi) = \operatorname{sen} t \quad t - \pi = v \Rightarrow f(v) = \operatorname{sen}(v + \pi) = -\operatorname{sen} v$

$$\mathcal{L}\{u_{3\pi}(t) \operatorname{sen} t\} = e^{-3\pi s} \mathcal{L}\{-\operatorname{sen} t\} = -e^{-3\pi s} \frac{1}{s^2+1}$$

$f(t - 3\pi) = \operatorname{sen} t \quad t - 3\pi = v \Rightarrow f(v) = \operatorname{sen}(v + 3\pi) = -\operatorname{sen} v$ $\boxed{2}$

$$\mathcal{L}\{u_{3\pi}(t)(t - 3\pi)\} = e^{-3\pi s} \mathcal{L}\{t\} = e^{-3\pi s} \frac{1}{s^2}$$

$f(t - 3\pi) = t - 3\pi \Rightarrow f(t) = t$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{3}{s} - \frac{3e^{-\pi s}}{s} - \frac{e^{-\pi s}}{s^2+1} - \frac{e^{-3\pi s}}{s^2+1} + \frac{e^{-3\pi s}}{s^2}}$$

$\boxed{2}$

D1 $\mathcal{L}^{-1}\left\{\ln\left(\frac{s-3}{s+1}\right)\right\} = \mathcal{L}^{-1}(F(s)) = f(t)$

Usando: $\mathcal{L}^{-1}\{F'(s)\} = -t f(t)$

$\boxed{2}$

$$F(s) = \ln\left(\frac{s-3}{s+1}\right)$$

$$F'(s) = \frac{1}{\left(\frac{s-3}{s+1}\right)} \cdot \left(\frac{s-3}{s+1}\right)'$$ regra da cadeia.

$$= \frac{s+1}{s-3} \cdot \frac{s+1-(s-3)}{(s+1)^2} = \frac{4}{(s-3)(s+1)}$$ $\boxed{4}$

$$\mathcal{L}^{-1}\{F'(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{(s-3)(s+1)}\right\} = (*)$$

$$\frac{4}{(s-3)(s+1)} = \frac{A}{s-3} + \frac{B}{s+1} = \frac{A(s+1)+B(s-3)}{(s-3)(s+1)} \Rightarrow$$

$A+B=0$ $\quad 4A=4 \quad A=1$

$A-3B=4$ $\quad B=-1$

$$(*) = \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \mathcal{L}^{-1}\left\{\frac{-1}{s+1}\right\} = e^{3t} - e^{-t}$$ $\boxed{2}$

$$\mathcal{L}^{-1}\{F'(s)\} = -t f(t) = e^{3t} - e^{-t}$$

$\boxed{2}$

$$\Rightarrow \boxed{f(t) = \frac{e^{-t} - e^{3t}}{t}}$$

D2 $\quad t x'' + (3t-1)x' + 3x = 0 \qquad x(0)=0$

$\downarrow \mathcal{L}$

$\mathcal{L}\{tx''\} + 3\mathcal{L}\{tx'\} - \mathcal{L}\{x'\} - 3\mathcal{L}\{x\} = 0.$ $\boxed{2}$

usar: $-F'(s) = \mathcal{L}\{tf(t)\}$

$= -[s^2X(s) - s\cancel{x(0)} - x'(0)]' - 3[sX(s) - \cancel{x(0)}]'$

$- sX(s) + \cancel{x(0)} + 3X(s)$

$= -[2sX(s) + s^2X'(s)] - 3\cancel{X(s)} - 3sX'(s)$

$-sX(s) + 3\cancel{X(s)} = 0$ (mult. por (-1) a eq.) $\boxed{2}$

$(s^2+3s)X'(s) + 3sX(s) = 0$

$X'(s) + \dfrac{3s}{s^2+3s}X(s) = 0.$ $\boxed{2}$

fator integrante:

$\mu(s) = e^{\int \frac{3s}{s^2+3s}ds} = e^{\int \frac{3}{s+3}} = e^{3\ln(s+3)} = (s+3)^3$ $\boxed{2}$

$\dfrac{d}{ds}\left[(s+3)^3 X(s)\right] = 0$

integra $\downarrow$

$(s+3)^3X(s) = C \Rightarrow X(s) = \dfrac{C}{(s+3)^3}$

$\xrightarrow{\mathcal{L}^{-1}} x(t) = \dfrac{C}{2!}e^{-3t}t^2$ $\boxed{2}$