A0 $\quad f(x) = \dfrac{x}{(1+x^2)^2}$ $\qquad \dfrac{d}{dx}\dfrac{1}{1+x^2} = -(1+x^2)^{-2} \cdot 2x$ 2

A série de potências para 2

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty}(-x^2)^k = \sum_{k=0}^{\infty}(-1)^k x^{2k} \quad |x|<1$$

série geométrica $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k$ $\quad |x|<1$ 2

Derivando termo a termo

$$\frac{d}{dx}\frac{1}{1+x^2} = \frac{-2x}{(1+x^2)^2} =$$

$$= \sum_{k=1}^{\infty}(-1)^k 2k\, x^{2k-1}$$ 2

$$\sum_{k=1}^{\infty}(-1)^k k\, x^{2k-1} = -\frac{x}{(1+x^2)^2}$$

$$\boxed{\sum_{k=1}^{\infty}(-1)^{k+1} k\, x^{2k-1} = \frac{x}{(1+x^2)^2}}$$ 2

A1 $\quad f(x) = \int_0^x e^{-t^2} dt$ (2)

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$ Raio de convergencia $r=\infty$

$$e^{-t^2} = \sum_{k=0}^{\infty} \frac{(-t^2)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{k!}$$ (2)

Integrar termo a termo (2)

$$\int_0^x e^{-t^2} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_0^x t^{2k} dt =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{t^{2k+1}}{2k+1} \Big|_0^x = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^{2k+1}}{(2k+1)}$$ (2)

$$\downarrow$$

$$t - \frac{t^3}{3!\,3} + \frac{t^5}{5!\,5} \cdots \Big|_0^x =$$ No limite inferior é igual a zero.

$$= x - \frac{x^3}{3!\,3} + \frac{x^5}{5!\,5} \cdots - 0$$

A série de potências para

$$\boxed{\int_0^x e^{-t^2} dt = \sum_{k=0}^{\infty} \frac{(-1)^k \, x^{2k+1}}{k!\,(2k+1)}}$$ (2)

A2 $\quad f(x) = x^3 \arctan x^3$

A série para $\arctan x = \int \frac{1}{x^2+1}\,dx$ ②

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty} (-x^2)^k = \sum_{k=0}^{\infty} (-1)^k x^{2k} \quad *$$ ②

série geométrica $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ $\quad |x|<1$

$$\arctan x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$$ ②

integra termo a termo $*$

$$\arctan x^3 = \sum_{k=0}^{\infty} \frac{(-1)^k (x^3)^{2k+1}}{2k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+3}}{2k+1}$$ ②

$$x^3 \arctan x^3 = x^3 \sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+3}}{2k+1} = \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+6}}{2k+1}}$$

②

A3. $f(x) = \frac{1-x^2}{(1+x^2)^2}$

$$\frac{d}{dx}\frac{x}{(1+x^2)} = \frac{(1+x^2)-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$ 2

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty}(-x^2)^k = \sum_{k=0}^{\infty}(-1)^k x^{2k}$$ 2

2 série geométrica $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ $|x|<1$

$$\frac{x}{1+x^2} = \sum_{k=0}^{\infty}(-1)^k x^{2k+1} = x - x^3 + x^5 \ldots$$

$\frac{d}{dx}$ deriva termo a termo 2

$$\boxed{\frac{1-x^2}{(1+x^2)^2} = \sum_{k=0}^{\infty}(-1)^k(2k+1)x^{2k}} = 1-3x^2+5x^4\ldots$$

2

30 $\quad X'(t) = \begin{pmatrix} 4 & 4 \\ 1 & 4 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 4e^{6t} \end{pmatrix}$ 2

a) $X'(t) = \begin{pmatrix} 4 & 4 \\ 1 & 4 \end{pmatrix} X(t)$

autovalores: $\det \begin{pmatrix} 4-\lambda & 4 \\ 1 & 4-\lambda \end{pmatrix} = 0 = (4-\lambda)^2 - 4 = \lambda^2 - 8\lambda + 12 \quad \lambda = \dfrac{8 \pm \sqrt{64-48}}{2}$

$\lambda = 6 \quad \lambda = 2$

autovetores $\lambda = 6 \quad \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad v_1 - 2v_2 = 0 \quad v_1 = 2 \quad v_2 = 1$ 2

$\lambda = 2 \quad \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad v_1 + 2v_2 = 0 \quad v_1 = 2 \Rightarrow v_2 = -1$

$X_1(t) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{6t} \quad X_2(t) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{2t} \Rightarrow \boxed{\Phi(t) = \begin{pmatrix} 2e^{6t} & 2e^{2t} \\ e^{6t} & -e^{2t} \end{pmatrix}}$

$X_c(t) = \Phi(t) \cdot C \qquad \boxed{C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}$ 2

(b) A solução particular $X_p(t) = \Phi(t) \cdot U(t)$
Pelo método de variação de parâmetros. 2

$U'(t) = \Phi^{-1}(t) F$

$\det \Phi(t) = -4e^{8t} \qquad \Phi^{-1}(t) = \dfrac{1}{-4e^{8t}} \begin{pmatrix} -e^{2t} & -2e^{2t} \\ -e^{6t} & 2e^{6t} \end{pmatrix}$

$\hookrightarrow U'(t) = \begin{pmatrix} \dfrac{e^{-6t}}{4} & \dfrac{e^{-6t}}{2} \\ \dfrac{e^{-2t}}{4} & -\dfrac{e^{-2t}}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 4e^{6t} \end{pmatrix} = \begin{pmatrix} 2 \\ -2e^{4t} \end{pmatrix} \xRightarrow{\text{integra.}}$

$U(t) = \begin{pmatrix} 2t + k_1 \\ -\dfrac{2e^{4t}}{4} + k_2 \end{pmatrix}$ Podemos assumir $k_1 = k_2 = 0$ pois aparece em $\Phi \cdot C$

2

$\boxed{U(t) = \begin{pmatrix} 2t \\ -\dfrac{e^{4t}}{2} \end{pmatrix}} \qquad \boxed{X_p(t) = \Phi(t) \cdot U(t)}$

3) $X'(t) = \begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix} X(t) + \begin{pmatrix} 13e^{14t} \\ 0 \end{pmatrix}$

a) $X'(t) = \begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix} X(t)$ $\quad \boxed{X_c = \phi(t) \cdot C \quad C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}$

autovalores $\det \begin{pmatrix} 2-\lambda & 4 \\ 3 & 13-\lambda \end{pmatrix} = (2-\lambda)(13-\lambda) - 12 = \lambda^2 - 15\lambda + 14 = 0$

$\lambda = \dfrac{15 \pm \sqrt{15^2 - 56}}{2} = \dfrac{15 \pm \sqrt{169}}{2} \begin{matrix} \nearrow 14 \\ \searrow 1 \end{matrix}$ (13)

autovetores

$\lambda = 1 \quad \begin{pmatrix} 1 & 4 \\ 3 & 12 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad v_1 + 4v_2 = 0 \quad v_1 = 4 \quad v_2 = -1$

$\lambda = 14 \quad \begin{pmatrix} -12 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad 3v_1 - v_2 = 0 \quad v_1 = 1 \quad v_2 = 3$

$X_1(t) = \begin{pmatrix} 4 \\ -1 \end{pmatrix} e^{t} \quad X_2(t) = \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{14t} \Rightarrow \boxed{\phi(t) = \begin{pmatrix} 4e^{t} & e^{14t} \\ -e^{t} & 3e^{14t} \end{pmatrix}}$

$\det \Phi(t) = 13e^{15t}$

(b) A solução particular é $\boxed{X_p(t) = \Phi(t) \cdot u(t)}$

Pelo método de variação de parametros

$u'(t) = \Phi^{-1}(t) F \Rightarrow \Phi^{-1}(t) = \dfrac{1}{13e^{15t}} \begin{pmatrix} 3e^{14t} & -e^{14t} \\ +e^{t} & 4e^{t} \end{pmatrix}$

$\downarrow$

$u'(t) = \begin{pmatrix} \frac{3}{13} e^{-t} & -\frac{1}{13} e^{-t} \\ \frac{1}{13} e^{-14t} & \frac{4}{13} e^{-14t} \end{pmatrix} \begin{pmatrix} 13e^{14t} \\ 0 \end{pmatrix} = \begin{pmatrix} 3e^{13t} \\ 1 \end{pmatrix} \xrightarrow{\text{integra.}}$

$u(t) = \begin{pmatrix} \dfrac{3e^{13t}}{13} + k_1 \\ t + k_2 \end{pmatrix}$

Podemos assumir $k_1 = k_2 = 0$.

$$\boxed{u(t) = \begin{pmatrix} \frac{3}{13} e^{13t} \\ t \end{pmatrix}}$$

B2

$$X' = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} X + \begin{pmatrix} 3e^{t} \\ 3e^{-t} \end{pmatrix}$$

2

a) $X'(t) = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} X(t)$

auto valores $\det \begin{pmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{pmatrix} = (1-\lambda)(3-\lambda) - 8 = \lambda^2 - 4\lambda - 5 = 0$

$\lambda = \frac{4 \pm \sqrt{16+20}}{2} \rightarrow \lambda = 5$, $\lambda = -1$

autovetores:

$\lambda = 5$ $\begin{pmatrix} -4 & 4 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $2v_1 - 2v_2 = 0$ $v_1 = v_2 = 1$

2 $\lambda = -1$ $\begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $2v_1 + 4v_2 = 0$ $v_1 = 2$ $v_2 = -1$

$X_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$ $X_2(t) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-t}$ $\Rightarrow$ $\boxed{\Phi(t) = \begin{pmatrix} e^{5t} & 2e^{-t} \\ e^{5t} & -e^{-t} \end{pmatrix}}$

2

$\boxed{X_c(t) = \Phi(t) \cdot C \quad C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}$

b) A solução particular $\boxed{X_p(t) = \Phi(t) \cdot u(t)}$

Pelo método de variação de parâmetros:

$\boxed{u'(t) = \Phi^{-1}(t) F}$ $\Rightarrow$ $\Phi^{-1}(t) = \frac{1}{-3e^{4t}} \begin{pmatrix} -e^{-t} & -2e^{-t} \\ -e^{5t} & e^{5t} \end{pmatrix}$

$\det \Phi = -3e^{4t}$

$$u'(t) = \begin{pmatrix} \frac{e^{-5t}}{3} & \frac{2}{3}e^{-5t} \\ \frac{e^{t}}{3} & -\frac{e^{t}}{3} \end{pmatrix} \begin{pmatrix} 3e^{t} \\ 3e^{-t} \end{pmatrix} = \begin{pmatrix} e^{-4t} + 2e^{-6t} \\ e^{2t} - 1 \end{pmatrix} \xrightarrow{\text{integ}}$$

2

$$\boxed{u(t) = \begin{pmatrix} \frac{e^{-4t}}{-4} + \frac{2e^{-6t}}{-6} \\ \frac{e^{2t}}{2} - t \end{pmatrix}}$$

2

B3 $X'(t) = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} X(t) + \begin{pmatrix} 8e^{7t} \\ 8 \end{pmatrix}$

2

a) $X'(t) = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} X(t)$

autovalores $\det \begin{pmatrix} 2-\lambda & 5 \\ 3 & 4-\lambda \end{pmatrix} = (2-\lambda)(4-\lambda) - 15 = \lambda^2 - 6\lambda - 7 = 0$

$\lambda = \dfrac{6 \pm \sqrt{36+28}}{2} = 3 \pm 4 \begin{matrix} \nearrow 7 \\ \searrow -1 \end{matrix}$

autovetores $\lambda = 7$ $\begin{pmatrix} -5 & 5 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $3v_1 - 3v_2 = 0$ $v_1 = v_2 = 1$

2

$\lambda = -1$ $\begin{pmatrix} 3 & 5 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $3v_1 + 5v_2 = 0$ $v_1 = 5$ $v_2 = -3$

$X_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{7t}$ $X_2(t) = \begin{pmatrix} 5 \\ -3 \end{pmatrix} e^{-t}$

$\boxed{\Phi(t) = \begin{pmatrix} e^{7t} & 5e^{-t} \\ e^{7t} & -3e^{-t} \end{pmatrix}}$

2 $\boxed{X_c(t) = \Phi(t) \cdot C \quad C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}$

$\det \Phi(t) = -8e^{6t}$

2

(b) A solução particular $X_p(t) = \Phi(t)\, u(t)$

Pelo método de variação de parâmetros

$\boxed{u'(t) = \Phi^{-1}(t) F}$ $\Phi^{-1}(t) = \dfrac{1}{-8e^{6t}} \begin{pmatrix} -3e^{-t} & -5e^{-t} \\ -e^{7t} & e^{7t} \end{pmatrix}$

$\downarrow$

$u'(t) = \begin{pmatrix} \frac{3}{8}e^{-7t} & \frac{5}{8}e^{-7t} \\ \frac{1}{8}e^{t} & -\frac{1}{8}e^{t} \end{pmatrix} \begin{pmatrix} 8e^{7t} \\ 8 \end{pmatrix} = \begin{pmatrix} 3 + 5e^{-7t} \\ e^{8t} - e^{t} \end{pmatrix}$

integra. $\rightarrow$ $\boxed{u(t) = \begin{pmatrix} 3t + \dfrac{5e^{-7t}}{-7} \\ \dfrac{e^{8t}}{8} - e^{t} \end{pmatrix}}$

Podemos assumir $k_1 = k_2 = 0$.

2

CO $(1-x^2)y'' - 5xy' - 4y = 0$ (2)

$y''(x) + P(x)y'(x) + Q(x)y(x) = 0 \Rightarrow P(x) = \frac{-5x}{1-x^2}$ $\quad Q(x) = \frac{-4}{1-x^2}$

$\lim_{x\to 0} P(x) = 0$ $\quad \lim_{x\to 0} Q(x) = -4$ Como ambos os limites existem $\Rightarrow$ $x=0$ é ponto ordinário

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \Rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \Rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1)x^{k-2}$

Substituindo: (2)

$$\sum_{k=2}^{\infty} c_k k(k-1)x^{k-2} - \sum_{k=2}^{\infty} c_k k(k-1)\underbrace{x^{k-2}x^2}_{x^k} - \sum_{k=1}^{\infty} 5c_k k \underbrace{x^{k-1}x}_{x^k} - \sum_{k=0}^{\infty} 4c_k x^k = 0$$

deslocamento $k \to k+2$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)x^k - \sum_{k=0}^{\infty} c_k k(k-1)x^k - \sum_{k=0}^{\infty} 5c_k k x^k - \sum_{k=0}^{\infty} 4c_k x^k = 0$$ (2)

pois $c_k k(k-1) = 0$ para $k=0$ e $k=1$

pois $5c_k k = 0$ para $k=0$

$$\sum_{k=0}^{\infty} \underbrace{\{c_{k+2}(k+2)(k+1) - [k(k-1)+5k+4]c_k\}}_{=0 \text{ pelo Princípio da Identidade}} x^k = 0$$

$$c_{k+2} = \frac{(\overbrace{k^2+4k+4}^{(k+2)^2})c_k}{(k+2)(k+1)} \Rightarrow \boxed{c_{k+2} = \frac{k+2}{k+1}c_k} \quad k \geq 0$$ (2)

$k=0$ $\quad c_2 = \frac{2}{1}c_0$

$k=1$ $\quad c_3 = \frac{3}{2}c_1$

$k=2$ $\quad c_4 = \frac{4}{3}c_2 = \frac{4}{3}\cdot\frac{2}{1}c_0$

$k=3$ $\quad c_5 = \frac{5}{4}c_3 = \frac{5}{4}\frac{3}{2}c_1$

$k=4$ $\quad c_6 = \frac{6}{5}c_4 = \frac{6}{5}\frac{4}{3}\frac{2}{1}c_0$

$k=5$ $\quad c_7 = \frac{7}{6}c_5 = \frac{7}{6}\frac{5}{4}\frac{3}{2}c_1$

$n \geq 1$ $\quad \boxed{c_{2n} = \frac{2n(2n-2)\ldots 6 4 2\, c_0}{(2n-1)(2n-3)\ldots 5 3 1}}$ $\quad \boxed{c_{2n+1} = \frac{(2n+1)(2n-1)\ldots 7 5 3\, c_1}{2n(2n-2)\ldots 6 4 2}}$

optativo $\Rightarrow c_{2n} = \frac{2^n n!\, 2^{n-1}(n-1)!}{(2n-1)!}c_0$ $\quad c_{2n+1} = \frac{(2n+1)!}{4^n(n!)^2}c_1$ (2)

$$y(x) = c_0 + \sum_{n=1}^{\infty} c_{2n}x^{2n} + c_1 x + \sum_{n=1}^{\infty} c_{2n+1}x^{2n+1}$$

C1

$$y'' - xy = 0 \Rightarrow y(x) = c_0 + \sum_{n=1}^{\infty} c_{3n} x^{3n} + c_1 x + \sum_{n=1}^{\infty} c_{3n+1} x^{3n+1}$$

$x=0$ é ponto ORDINÁRIO pois para

$y'' + P(x)y' + Q(x)y = 0$ temos $P(x) = 0$ e $Q(x) = -x$

$\lim_{x\to 0} P(x) = 0$ e $\lim_{x\to 0} Q(x) = 0$ 2

Como os limites existem $\Rightarrow$ $x=0$ é pto ordinário

$$y(x) = \sum_{k=0}^{\infty} c_k x^k \Rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1}$$

$$y''(x) = \sum_{k=2}^{\infty} c_k k(k-1) x^{k-2}$$ 2

$$\sum_{k=2}^{\infty} c_k k(k-1) x^{k-2} - \sum_{k=0}^{\infty} c_k x^{k+1} = 0$$

deslocamentos $k \to k+2$ $\quad$ $k \to k-1$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1) x^k - \sum_{k=1}^{\infty} c_{k-1} x^k = 0$$ 2

$$\underbrace{c_2 \cdot 2 \cdot 1\, x^0}_{k=0} + \sum_{k=1}^{\infty} \left(c_{k+2}(k+2)(k+1) - c_{k-1}\right) x^k = 0$$

Relação de recorrência: 2 $\boxed{c_{k+2} = \dfrac{c_{k-1}}{(k+2)(k+1)}}$ $k \geq 1$

Pelo princípio da identidade 2

$\Rightarrow \boxed{c_2 = 0}$

$k=1$ $\quad c_3 = \dfrac{c_0}{3.2}$ $\quad$ | $\quad k=2$ $\quad c_4 = \dfrac{c_1}{4.3}$ $\quad$ $k=3$ $\quad c_5 = \dfrac{c_2}{5.4} = 0$

$k=4$ $\quad c_6 = \dfrac{c_3}{6.5} = \dfrac{c_0}{6532}$ $\quad$ | $\quad k=5$ $\quad c_7 = \dfrac{c_4}{7.6} = \dfrac{c_1}{7.6.4 3}$

$k=6$ $\quad c_8 = \dfrac{c_5}{*} = 0 \Rightarrow \boxed{c_{3n+2} = 0}$

$k=7$ $\quad c_9 = \dfrac{c_6}{9.8} = \dfrac{c_0}{986532} \Rightarrow$ $\quad c_{3n} = \dfrac{c_0}{(3n.(3n-3)\ldots 963)(3n-1\ldots 852)}$

$k=8$ $\quad c_{10} = \dfrac{c_7}{10.9} = \dfrac{c_1}{10.9\,7643} \Rightarrow$ $\quad c_{3n+1} = \dfrac{c_1}{[(3n+1).(3n-2)\ldots 10.7.4][3n\ldots 9.6\ldots}$

C2 $(x^2-4)y'' - xy' - 3y = 0$ (2)

$y'' + P(x)y' + Q(x)y = 0 \Rightarrow P(x) = \frac{-x}{x^2-4}$ e $Q(x) = \frac{-3}{x^2-4}$

Como $\lim_{x\to 0} P(x) = 0$ e $\lim_{x\to 0} Q(x) = 3/4$ ambos existem $\Rightarrow$

$x=0$ é ponto ordinário

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \Rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \Rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1) x^{k-2}$ (2)

Substituindo:

$\sum_{k=2}^{\infty} c_k k(k-1) \underbrace{x^{k-2} x^2}_{\to x^k} - \sum_{k=2}^{\infty} 4c_k k(k-1)x^{k-2} - \sum_{k=1}^{\infty} c_k k \underbrace{x^{k-1} x}_{\to x^k} - \sum_{k=0}^{\infty} 3c_k x^k = 0$

deslocamento $k \to k+2$

$\sum_{k=0}^{\infty} c_k k(k-1)x^k - \sum_{k=0}^{\infty} 4c_{k+2}(k+2)(k+1)x^k - \sum_{k=0}^{\infty} c_k k x^k - \sum_{k=0}^{\infty} 3c_k x^k = 0$ (2)

pois $c_k k(k-1) = 0$ para $k=0$ e $k=1$

pois $c_k k = 0$ para $k=0$

$\sum_{k=0}^{\infty} \{\underbrace{[k(k-1) - k - 3]c_k - [4(k+2)(k+1)]c_{k+2}}_{=0 \text{ pelo Princípio da Identidade}}\} x^k = 0$

$k^2 - 2k - 3 = 0$

$\frac{2 \pm \sqrt{4+12}}{2} \nearrow 3, \searrow -1$

$\to = (k-3)(k+1)$

$\Rightarrow$ Relação de Recorrência: $k \geq 0$

$c_{k+2} = \frac{k(k-1) - k - 3}{4(k+2)(k+1)} c_k$ (2)

$\to c_{k+2} = \frac{(k-3)\cancel{(k+1)}}{4(k+2)\cancel{(k+1)}} c_k$

$k=1$

$c_3 = \frac{-2\, c_1}{4.3}$ (2)

$k=3$ $\quad c_5 = 0$

$\Rightarrow c_{2n+1} = 0$, $n \geq 2$

$k=0 \quad c_2 = \frac{-3}{4.2} c_0$

$k=2 \quad c_4 = \frac{(-1).c_2}{4.4} = \frac{(-1)(-3)c_0}{4.4\;4.2}$

$k=4 \quad c_6 = \frac{1.c_4}{4.6} = \frac{1.(-1)(-3)c_0}{4.6\;4.4\;4.2}$

$k=6 \quad c_8 = \frac{3\,c_6}{4.8} = \frac{3\;1.(-1).(-3)c_0}{4.8\;4.6\;4.4\;4.2}$

$n \geq 3 \quad c_{2n} = \frac{3.[(2n-5)(2n-7)\dots 3\;1]\,c_0}{4^n[2n.(2n-2)\dots 4.2]}$

$y(x) = c_0\left(1 - \frac{3}{8}x^2 + \frac{3}{128}x^4 + \sum_{n=3}^{\infty} \frac{3[(2n-5)(2n-7)\dots 3.1]\,x^{2n}}{4^n[2n.(2n-2)\dots 4.2]}\right) +$

$+ c_1\left(x - \frac{1}{6}x^3\right)$

(3 $y'' + 2xy' + 3y = 0$ $\quad y'' + P(x)y' + Q(x)y = 0 \Rightarrow$ (2)

$P(x) = 2x$ e $Q(x) = 3$. Como $\lim_{x\to 0} P(x) = 0$ e $\lim_{x \to 0} Q(x) = 3$ existem $\Rightarrow$ $x = 0$ é ponto ordinário

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \Rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \Rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1)x^{k-2}$ (2)

Substituindo

$$\sum_{k=2}^{\infty} c_k k(k-1)x^{k-2} + \sum_{k=1}^{\infty} 2c_k k \underbrace{x^{k-1}x}_{x^k} + \sum_{k=0}^{\infty} 3c_k x^k = 0$$

desbocamento $k \to k+2$ $\quad$ $k=0$ pois $c_k k = 0$ para $k=0$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)x^k + \sum_{k=0}^{\infty} 2c_k k x^k + \sum_{k=0}^{\infty} 3c_k x^k = 0$$ (2)

$$\sum_{k=0}^{\infty} \underbrace{\left[c_{k+2}(k+2)(k+1) + (2k+3)c_k\right]}_{=0} x^k = 0$$

pelo Princípio da Ident. $\Rightarrow$

(2) Fórmula de recorrência: $\boxed{c_{k+2} = \dfrac{-(2k+3)c_k}{(k+2)(k+1)}}$ $\quad k \geq 0$

$k=0$ $\quad c_2 = -\dfrac{3c_0}{2.1}$ $\quad \longrightarrow \quad$ $k=1$ $\quad c_3 = -\dfrac{(2.1+3)c_1}{3.2}$ $\longrightarrow$

$k=2$ $\quad c_4 = -\dfrac{(2.2+3)c_2}{4.3} = -\dfrac{(2.2+3)(-3)c_0}{4.3\ 2\ 1}$ $\longrightarrow$

$\longrightarrow$ $k=3$ $\quad c_5 = \dfrac{-(2.3+3)c_3}{5.4} = \dfrac{-(2.3+3)[-(2.1+3)]c_1}{5\,4\ 3\,2}$

$k=4$ $\quad c_6 = -\dfrac{(2.4+3)c_4}{6\,5} = \dfrac{-(2.4+3)[-(2.2+3)][-3]c_0}{6\,5\ .4.3\,2\,1}$

$$\boxed{c_{2n} = \frac{(-1)^n [2.(2n-2)+3][4n-5] \ldots 11.7.3}{(2n)!} c_0 \quad n \geq 1}$$

$$\boxed{c_{2n+1} = \frac{(-1)^n [2.(2n-1)+3][4n-3] \ldots 9\,5}{(2n+1)!} c_1 \quad n \geq 1}$$ (2)

Solução:

$$y(x) = c_0 + \sum_{n=1}^{\infty} c_{2n} x^{2n} + c_1 x + \sum_{n=1}^{\infty} c_{2n+1} x^{2n+1}$$

DO

$$f(x) = \begin{cases} -2 & 0 \leq x < \pi \\ 3 & -\pi \leq x < 0 \end{cases} \qquad f(x+2\pi) = f(x)$$

2

$$P = 2L = 2\pi \Rightarrow L = \pi$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 3\,dx + \int_{0}^{\pi} -2\,dx\right] = \frac{1}{\pi}\left[3x\Big|_{-\pi}^{0} - 2x\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-(-3\pi) - (2\pi - 0)\right] = \frac{\pi}{\pi} = 1 \qquad \boxed{a_0 = 1}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 3\cos nx + \int_{0}^{\pi} -2\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[3\,\frac{\operatorname{sen} nx}{n}\Big|_{-\pi}^{0} - 2\,\frac{\operatorname{sen} nx}{n}\Big|_{0}^{\pi}\right] = \frac{1}{\pi}\Big[3\Big(\underbrace{\operatorname{sen} 0}_{0} - \underbrace{\frac{\operatorname{sen}(-n\pi)}{n}}_{0}\Big)$$

2

$$-2\Big(\underbrace{\frac{\operatorname{sen} n\pi}{n}}_{0} - \underbrace{\operatorname{sen} 0}_{0}\Big)\Big] = 0 \Rightarrow \boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\operatorname{sen} nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 3\operatorname{sen} nx\,dx + \int_{0}^{\pi} -2\operatorname{sen} nx\,dx\right]$$

$$\frac{1}{\pi}\left[-3\,\frac{\cos nx}{n}\Big|_{-\pi}^{0} + 2\,\frac{\cos nx}{n}\Big|_{0}^{\pi}\right] = \frac{1}{\pi}\Big[\Big(-\frac{3}{n} + \frac{3}{n}\underbrace{\cos(-n\pi)}_{\cos n\pi = (-1)^n}\Big)$$

$$+\Big(\frac{2}{n}\underbrace{\cos n\pi}_{=(-1)^n} - \frac{2}{n}\Big)\Big] = \left[-\frac{5}{n\pi} + (-1)^n\left(\frac{3}{n\pi} + \frac{2}{n\pi}\right)\right] \Rightarrow$$

2

$$\boxed{b_n = \frac{5}{n\pi}\left((-1)^n - 1\right)}$$

$$\boxed{f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{5}{n\pi}\left((-1)^n - 1\right)\operatorname{sen} nx}$$

2

2 $\frac{a_0}{2}$.

b) A série de Fourier converge em $x = \pi$ para $\frac{f(\pi^+) + f(\pi^-)}{2} = \frac{3 + (-2)}{2} = \frac{1}{2}$ pelo Teorema de Conv. de Fourier

D1

$$f(x) = \begin{cases} -3 & 0 \le x < 4 \\ 1 & -4 \le x < 0 \end{cases} \qquad f(x+8) = f(x)$$

$P = 2L = 8 \qquad L = 4$

2

$$a_0 = \frac{1}{4}\int_{-4}^{4} f(x)\,dx = \frac{1}{4}\left[\int_{-4}^{0} dx + \int_0^4 -3\,dx\right] =$$

$$= \frac{1}{4}\left[x\Big|_{-4}^{0} - 3x\Big|_0^4\right] = \frac{1}{4}\left[0-(-4)-(12-0)\right] = -\frac{8}{4} = -2$$

$$a_n = \frac{1}{4}\int_{-4}^{4} f(x)\cos\frac{n\pi x}{4}\,dx = \frac{1}{4}\left[\int_{-4}^{0}\cos\frac{n\pi x}{4}\,dx + \int_0^4 -3\cos\frac{n\pi x}{4}\,dx\right]$$

2

$$\frac{1}{4}\left[\frac{4}{n\pi}\operatorname{sen}\frac{n\pi x}{4}\Big|_{-4}^{0} - \frac{3.4}{n\pi}\operatorname{sen}\frac{n\pi x}{4}\Big|_0^4\right] =$$

$$\frac{1}{n\pi}\Big[\big(\underbrace{\operatorname{sen} 0}_{0} - \underbrace{\operatorname{sen}(-n\pi)}_{0}\big) - 3\big(\underbrace{\operatorname{sen} n\pi}_{0} - \underbrace{\operatorname{sen} 0}_{0}\big)\Big] = 0$$

$$b_n = \frac{1}{4}\int_{-4}^{4} f(x)\operatorname{sen}\frac{n\pi x}{4}\,dx = \frac{1}{4}\left[\int_{-4}^{0}\operatorname{sen}\frac{n\pi x}{4}\,dx + \int_0^4 (-3)\operatorname{sen}\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{4}\left[-\frac{4}{n\pi}\cos\frac{n\pi x}{4}\Big|_{-4}^{0} + \frac{3.4}{n\pi}\cos\frac{n\pi x}{4}\Big|_0^4\right]$$

2

$$= \frac{1}{n\pi}\Big[\big(-\underbrace{\cos 0}_{1} + \underbrace{\cos(-n\pi)}_{(-1)^n}\big) + 3\big(\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cos 0}_{1}\big)\Big] =$$

$$\frac{1}{n\pi}\left[4(-1)^n - 4\right] = \frac{4}{n\pi}\left((-1)^n - 1\right)$$

2

$$\boxed{f(x) = \underbrace{-\frac{\cancel{2}}{\cancel{2}}}_{-1} + \sum_{n=1}^{\infty} \frac{4}{n\pi}\left((-1)^n - 1\right)\operatorname{sen}\frac{n\pi x}{4}}$$

$a_0/2$

2

(b) A série de Fourier converge em $x=-4$ para

$$\frac{f(-4^+) + f(-4^-)}{2} = \frac{1 + (-3)}{2} = \frac{-2}{2} = -1$$

pelo Teorema de Convergência de Fourier

D2

$$f(x) = \begin{cases} -5 & 0 \leq x < 2 \\ 1 & -2 \leq x < 0 \end{cases} \qquad f(x+4) = f(x)$$

2

$$P = 2L = 4 \Rightarrow L = 2$$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\left[\int_{-2}^{0} dx + \int_{0}^{2} -5\,dx\right] =$$

$$\frac{1}{2}\left[x\Big|_{-2}^{0} - 5x\Big|_{0}^{2}\right] = \frac{1}{2}\left[(0-(-2)) - (10-0)\right] = \frac{-8}{2} = -4$$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}\,dx =$$

$$\frac{1}{2}\left[\int_{-2}^{0}\cos\frac{n\pi x}{2} + \int_{0}^{2}(-5)\cos\frac{n\pi x}{2}\,dx\right] = \frac{1}{2}\left[\frac{2}{n\pi}\,\text{sen}\frac{n\pi x}{2}\Big|_{-2}^{0} - \frac{5.2}{n\pi}\,\text{sen}\frac{n\pi x}{2}\Big|_{0}^{2}\right]$$

$$= \frac{1}{n\pi}\left[(\underbrace{\cancel{\text{sen}}\,0}_{0} - \underbrace{\text{sen}(-\cancel{n}\pi)}_{0}) - 5(\underbrace{\text{sen}\,\cancel{n}\pi}_{0} - \underbrace{\cancel{\text{sen}}\,0}_{0})\right] = 0$$

2

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\,\text{sen}\frac{n\pi x}{2}\,dx = \frac{1}{2}\left[\int_{-2}^{0}\text{sen}\frac{n\pi x}{2}\,dx + \int_{0}^{2}(-5)\,\text{sen}\frac{n\pi x}{2}\,dx\right]$$

$$= \frac{1}{2}\left[-\frac{2}{n\pi}\cos\frac{n\pi x}{2}\Big|_{-2}^{0} + \frac{5 \cdot 2}{n\pi}\cos\frac{n\pi x}{2}\Big|_{0}^{2}\right] =$$

$$= \frac{1}{n\pi}\left[\left(-\underbrace{\cancel{\cos}\,0}_{1} + \underbrace{\cos(-n\pi)}_{(-1)^n}\right) + 5\left(\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cancel{\cos}\,0}_{1}\right)\right] =$$

$$= \frac{1}{n\pi}\left[6(-1)^n - 6\right] = \frac{6}{n\pi}\left((-1)^n - 1\right)$$

2

$$f(x) = \underbrace{\frac{-4}{2}}_{\frac{a_0}{2}} + \sum_{n=1}^{\infty}\frac{6}{n\pi}\left((-1)^n - 1\right)\text{sen}\frac{n\pi x}{2}$$

2

(b) A série de Fourier converge em $x=0$ para

$$\frac{f(0^+) + f(0^-)}{2} = \frac{-5+1}{2} = -2.$$

pelo Teorema de Convergência de Fourier

2

D3 $f(x) = \begin{cases} -2 & 0 \leq x < 5 \\ 1 & -5 \leq x < 0 \end{cases}$ $f(x+10) = f(x)$

2

$P = 2L = 10 \Rightarrow L = 5$

$$a_0 = \frac{1}{5}\int_{-5}^{5} f(x)\,dx = \frac{1}{5}\left[\int_{-5}^{0} dx + \int_{0}^{5} -2\,dx\right] = \frac{1}{5}\left[x\Big|_{-5}^{0} + (-2)x\Big|_{0}^{5}\right]$$

$$= \frac{1}{5}\left(\left(0-(-5)\right) + \left(-10+0\right)\right) = -\frac{5}{5} = -1$$

$$a_n = \frac{1}{5}\int_{-5}^{5} f(x)\cos\frac{n\pi x}{5} = \frac{1}{5}\left[\int_{-5}^{0}\cos\frac{n\pi x}{5}\,dx + \int_{0}^{5}(-2)\cos\frac{n\pi x}{5}\,dx\right] =$$

$$= \frac{1}{5}\left[\frac{5}{n\pi}\operatorname{sen}\frac{n\pi x}{5}\Big|_{-5}^{0} - 2\cdot\frac{5}{n\pi}\operatorname{sen}\frac{n\pi x}{5}\Big|_{0}^{5}\right] =$$

2

$$= \frac{1}{n\pi}\Big[\big(\underbrace{\operatorname{sen} 0}_{0} - \underbrace{\operatorname{sen}(-n\pi)}_{0}\big) - 2\big(\underbrace{\operatorname{sen} n\pi}_{0} - \underbrace{\operatorname{sen} 0}_{0}\big)\Big] = 0$$

$$b_n = \frac{1}{5}\int_{-5}^{5} f(x)\operatorname{sen}\frac{n\pi x}{5}\,dx = \frac{1}{5}\left[\int_{-5}^{0}\operatorname{sen}\frac{n\pi x}{5}\,dx + \int_{0}^{5}(-2)\operatorname{sen}\frac{n\pi x}{5}\,dx\right] =$$

$$= \frac{1}{5}\left[-\frac{5}{n\pi}\cos\frac{n\pi x}{5}\Big|_{-5}^{0} + 2\cdot\frac{5}{n\pi}\cos\frac{n\pi x}{5}\Big|_{0}^{5}\right] =$$

$$\frac{1}{n\pi}\Big[-\underbrace{\cos 0}_{1} + \underbrace{\cos(-n\pi)}_{(-1)^n}\big) + 2\big(\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cos 0}_{1}\big)\Big] =$$

2

$$\frac{1}{n\pi}\left[3\cdot(-1)^n - 3\right] = \frac{3}{n\pi}\left((-1)^n - 1\right)$$

$$\boxed{f(x) = -\underbrace{\frac{1}{2}}_{\frac{a_0}{2}} + \sum_{n=1}^{\infty}\frac{3}{n\pi}\left((-1)^n - 1\right)\operatorname{sen}\frac{n\pi x}{5}}$$

2

(b) A série de Fourier converge em $x = 0$ para $\frac{f(0^+) + f(0^-)}{2} = \frac{-2+1}{2} = -\frac{1}{2}$ pelo Teorema de Convergência de Fourier.

2