LISTA #2 – MS993/MT404 – 2S2016 – IMECC/UNICAMP
Matemática Aplicada

Atenção: A Lista #2 poderá ser feita em grupos de até no máximo 3 (três) estudantes.

Atividade	Temas, palavras-chave	Disponibilização	Entrega
Lista #2 (**L2**) David S. Watkins, Fundamentals of Matrix Computations, New Jersey: John Wiley & Sons (3 ed., 2010). Yousef Saad. Iterative methods for sparse linear systems, 2nd ed. Philadelphia, PA, SIAM (2003). Gene H. Golub and Charles F. Van Loan. Matrix computations, 3rd ed., Johns Hopkins University Press (1996).	Dense vs. sparse systems, Cayley-Hamilton theorem, Krylov spaces, Krylov methods (symmetric systems), Krylov methods (nonsymmetric systems), Matrices and their powers, eigenvalues, pseudospectrum; Arnoldi's method; Cayley-Hamilton theorem, Hessenberg matrices (Reducing a Matrix to Hessenberg Form); Minimal Residual Iteration, The Symmetric Lanczos Algorithm, Chebyshev Polynomials, Monic polynomial, minimal polynomial, Computations in floating point and exact arithmetic	23/Set	21/Out

List of exercises

Gene H. Golub and Charles F. Van Loan. Matrix computations, 3rd ed., Johns Hopkins University Press (1996).

- Prove Theorem 10.1.1 (page 511), but remembering to fill in all of the details required. Moreover, as before, explain how to use such result in practical applications.
- Prove Theorem 10.1.2 (page 512), but again remembering to fill in all of the details required. Moreover, explain how to use such result in practical applications.
- To do problems P10.1.2, P10.1.4 and P 10.1.6. (page 517)

Yousef Saad. Iterative methods for sparse linear systems, 2nd ed. Philadelphia, PA, SIAM (2003).

- Prove Theorem Theorem 4.1 (page 115), consulting other book and papers (if needed) and explain how to use such result in practical applications.
- Prove Corollary 4.2 (page 115), consulting other book and papers (if needed) and explain how to use such result in practical applications.
- Prove Theorem 4.4 (page 118), but remembering to fill in all of the details required. Moreover, explain how to use such result in practical applications. Do the same for the next questions.
- Prove Theorem 4.6 (page 120).
- Prove Theorem 4.7 (page 120).
- Prove Corollary 4.8.
- Prove Theorem 4.9 (page 121 and page 122).
- Prove Theorem 4.10 (page 122).
- Prove Proposition 4.12 (page 124).
- Prove Proposition 4.15 (page 125 and page 126).
- Prove Proposition 4.16 (page 126 and page 127).
- To do problems P-4.1 (page 130) and P-4.4 (page 131).

David S. Watkins, Fundamentals of Matrix Computations, New Jersey: John Wiley & Sons (3 ed., 2010).

- To do Exercise 8.2.18 (page 562). Explain how to use such result in practical applications.
- To do Exercise 8.2.27 (page 567) and Exercise 8.2.28 (page 568).
- To do Exercise 8.3.12 (page 573)

- To do Exercise 8.3.16 (page 575)
- Exercise 8.3.22 (page 577) and Exercise 8.3.23 (page 577)
- To do Exercise 8.3.27 (page 577)
- To do Exercise 8.3.41 (page 582)

GENERAL QUESTIONS

Remark: For the following itens, you might also consider the references (Take a look at the notes and references at page 132 in the book of Yousef Saad). “Richard S. Varga, Matrix iterative analysis, Springer series in computational mathematics; 27), 2000.” and “David M. Young, Iterative solution of large linear systems, Academic Press (Computer science and applied mathematics), 1971.”

1. Discuss a brief summary (maximum two pages) about ***i)*** The Chebyshev Semi-Iterative Method (Chebyshev polynomials) and ***ii)*** symmetric successive overrelaxation (SSOR) as two ways to accelerate the convergence of iterative methods, including appropriate references if needed.

2. Discuss (two page maximum) about the relevance of spectral radii of nonnegative matrices to the subsequent development of iterative methods as well as to their convergence properties. In particular, this theory provide us both nontrivial upper and lower bounds for the spectral radius for this class of matrices.

3. In this exercise you must derive (along with all details) the Conjugate Gradient (CG) and Preconditioned Conjugate Gradient (PCG) algorithms. Indeed, prove that in finite arithmetic both CG and PCG converge in n steps, which n is the dimension of the associated matrix A, linked to the linear system Ax=b (consider A SPD and a nonzero vector b). In addition, include the converge rates for each one of these algorithms (i.e. CG and PCG). *Here is very advisable to consult "A. Greenbaum and Z. Strakos. Predicting the Behavior of Finite Precision Lanczos and Conjugate Gradient Computations. SIAM. J. Matrix Anal. & Appl., 13(1), 121-137." In short, in this paper* ***<u>a subtle backward error analysis was devised</u>*** *to explain the observed behavior of Conjugate Gradient (CG) Computations in floating point and explain how it can differ from exact arithmetic.*

4. Read Chapters 5, 6 and 7 of the book “Iterative Methods for Sparse Linear Systems” (2nd edition) by Yousef Saad. Next, collect to key results in **<u>your own opinion</u>** about the a) Projection Methods and b) Krylov Subspace Methods. (There is no limit of pages for this exercise.)

<u>Additional references:</u>

- James W. Demmel. Applied numerical linear algebra, Philadelphia, PA, SIAM (1997).
- Lloyd N. Trefethen, David Bau III. Numerical linear algebra, Philadelphia, PA, SIAM (1997).
- Roger A. Horn and Charles R. Johnson. Matrix analysis, Cambridge, MA, Cambridge University Press (1985).
- Wolfgang Hackbusch. Iterative solution of large sparse systems of equations, New York, NY, Springer (1994).
- Richard Barrett, Michael W. Berry, Tony F. Chan, James Demmel, June Donato, Jack Dongarra, Victor Eijkhout, Roldan Pozo, Charles Romine, Henk van der Vorst, emplates for the Solution of Linear Systems: Building Blocks for Iterative Methods (Software, environments, tools) SIAM, 1994.
- Carl D. Meyer, Matrix Analysis and Applied Linear Algebra, Philadelphia, PA, SIAM (2000).
- David S. Watkins, Fundamentals of Matrix Computations, John Wiley & Sons (2 ed., 2002) e (3 ed., 2010).
- Gene H. Golub and Charles F. Van Loan. Matrix computations, 3rd ed., Johns Hopkins University Press (1996).
- Gérard Meurant. The Lanczos and conjugate gradient algorithms: from theory to finite precision computations, Philadelphia, PA, SIAM (2006).
- Henk A. van der Vorst. [recurso eletrônico, digital, PDF file] Iterative Krylov Methods for Large Linear Systems, Cambridge University Press - Monographs on applied and computational mathematics; n. 13, (2003).