AO $\sum_{n=1}^{\infty} \frac{(-1)^n}{n 5^n} (x-3)^n$ Usar o teste da Razão:

$$\lim_{n\to\infty} \frac{|x-3|^{n+1}}{(n+1)\, 5^{n+1}} \cdot \frac{n\, 5^n}{|x-3|^n} = \lim_{n\to\infty} |x-3| \frac{n}{n+1} \cdot \frac{1}{5}$$

$$= \frac{|x-3|}{5} \lim_{n\to\infty} \frac{n}{n+1} \to 1 = \frac{|x-3|}{5} < 1 \text{ converge.}$$

$$> 1 \text{ diverge}$$

4

$$|x-3| < 5 \Rightarrow -5 < x-3 < 5 \Rightarrow -2 < x < 8$$ 2

$x = -2$ $\sum \frac{(-1)^n (-5)^n}{n\, 5^n} = \sum \frac{(-1)^n (-1)^n}{n} \frac{5^n}{5^n} = \sum \frac{1}{n}$

diverge

2

$x = 8$ $\sum \frac{(-1)^n \cancel{5^n}}{n \cancel{5^n}} = \sum \frac{(-1)^n}{n}$ converge condicionalmente

2

intervalo de convergência.

$(-2, 8]$ i.e. $\boxed{-2 < x \leq 8}$

A1 $\sum_{n=1}^{\infty} \frac{1}{2n}(x-1)^n$ Usar o teste da Razão:

$$\lim_{n\to\infty} \frac{1}{2(n+1)} |x-1|^{n+1} \cdot \frac{2n}{|x-1|^n} = |x-1| \lim_{n\to\infty} \frac{2n}{2n+2}$$

Como $\lim_{n\to\infty} \frac{2n}{2n+2} = \lim_{n\to\infty} \frac{2}{2+2/n} = 1$

temos se $|x-1|<1$ a série converge

4 se $|x-1|>1$ a série diverge

2 $|x-1|<1 \Rightarrow -1 < x-1 < 1 \Rightarrow 0 < x < 2$

$x=0$ $\sum \frac{(-1)^n}{2n}$ converge condicionalmente

pois $\sum \frac{1}{n}$ diverge e pelo teste das séries alternadas $\left\{\frac{1}{2n}\right\}$ decresce e $\lim \frac{1}{2n} = 0$

2

$\Rightarrow \sum \frac{(-1)^n}{2n}$ converge.

$x=2$ $\sum \frac{1}{2n}$ diverge.

2

Intervalo de convergência.

$$\boxed{0 \leq x < 2}$$

A2 $\sum_{n=1}^{\infty} \frac{1}{2n+7} (x-2)^n$

Usar o teste da razão:

$\lim_{n\to\infty} \frac{1}{2(n+1)+7} |x-2|^{n+1} \cdot \frac{2n+7}{|x-2|^n} =$ 4

$= |x-2| \lim_{n\to\infty} \frac{2n+7}{2n+9} = |x-2| \lim_{n\to\infty} \frac{2+7/n}{2+9/n} =$

$|x-2| < 1$ a serie converge

> 1 a serie diverge

$|x-2|<1 \Rightarrow -1 < x-2 < 1 \Rightarrow 1 < x < 3$ 2

$x=1 \Rightarrow \sum \frac{(-1)^n}{2n+7}$ converge (condicionalmente)

2 • converge pelo teste das series alternadas $\left(\frac{1}{2n+7}\right)$ seq. decrescente c/ $\lim_{n\to\infty} \frac{1}{2n+7} = 0$

$x = 3$ $\sum \frac{1}{2n+7}$ diverge pelo teste limite - comparação c/ $\sum \frac{1}{n}$

2 $\lim_{n\to\infty} \frac{\frac{1}{2n+7}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2n+7} = \frac{1}{2} = \rho$

Como $\rho \neq 0$ e $\rho \neq \infty$ ambas tem o mesmo comportamento.

Neste caso divergem.

$\boxed{\text{Interv. de Convergência } 1 \leq x < 3}$

A3 $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}(x+2)^n$ 4

usar o teste da Razão:

$$\lim_{n\to\infty} \frac{1}{(n+1)3^{n+1}} |x+2|^{n+1} \cdot \frac{n3^n}{|x+2|^n} =$$

$$= \lim_{n\to\infty} \frac{|x+2|}{3} \cdot \frac{n}{n+1} = \frac{|x+2|}{3} \underbrace{\lim_{n\to\infty} \frac{n}{n+1}}_{1}$$

$$= \frac{|x+2|}{3} < 1 \quad \text{a série converge}$$

$$> 1 \quad \text{a série diverge}$$

$|x+2| < 3 \Rightarrow -3 < x+2 < 3 \Rightarrow$ 2

$\Rightarrow -5 < x < 1$

$x = -5$ $\sum \frac{(-1)^n}{n3^n}(-3)^n = \sum \frac{(-1)^n(-1)^n \cancel{3^n}}{n \cancel{3^n}} = \sum \frac{1}{n}$ diverge

2

$x = 1$ $\sum \frac{(-1)^n 3^n}{n 3^n} = \sum \frac{(-1)^n}{n}$ converge

pelo teste das séries alternadas:

pois $\left(\frac{1}{n}\right)$ seq. decrescente $\lim_{n\to\infty} \frac{1}{n} = 0$ 2

Intervalo de convergência

$$\boxed{-5 < x \leq 1}$$

80 $X' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} X + \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix}$

a) $X' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} X$

- $\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = 0 = (1-\lambda)(3-\lambda)+1 = \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2$

2

$\lambda = 2$ autovalor repetido.

- $(A - \lambda I) V = 0 \quad \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0 \quad V_1 + V_2 = 0$
$V_1 = 1 \Rightarrow V_2 = -1$

$X_1(t) = Ve^{\lambda t} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$

2 $(A - \lambda I) W = V \quad \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow w_1 + w_2 = -1 \quad w_1 = 1 \Rightarrow w_2 = -2$

$X_2(t) = \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} t + \begin{pmatrix} 1 \\ -2 \end{pmatrix}\right] e^{2t}$

$\boxed{X_c(t) = \Phi(t) \cdot C} \quad C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

2 $\Phi(t) = \begin{bmatrix} e^{2t} & (t+1)e^{2t} \\ -e^{2t} & -(t+2)e^{2t} \end{bmatrix} \Rightarrow \det \Phi = e^{4t}(-(t+2) + (t+1)) = -e^{4t}$

2 $\Phi^{-1} = \frac{1}{-e^{4t}} \begin{pmatrix} -(t+2)e^{2t} & -(t+1)e^{2t} \\ e^{2t} & e^{2t} \end{pmatrix} = \begin{pmatrix} +(t+2)e^{-2t} & (t+1)e^{-2t} \\ -e^{-2t} & -e^{-2t} \end{pmatrix}$

$U' = \Phi^{-1} \cdot F = \begin{pmatrix} (t+2)e^{-2t} & (t+1)e^{-2t} \\ -e^{-2t} & -e^{-2t} \end{pmatrix}\begin{pmatrix} 0 \\ e^{2t} \end{pmatrix} = \begin{pmatrix} t+1 \\ -1 \end{pmatrix}$

2 $U = \begin{pmatrix} \int t+1 \, dt \\ \int -dt \end{pmatrix} = \begin{pmatrix} \frac{t^2}{2} + t + k_1 \\ -t + k_2 \end{pmatrix}$ podemos supor $k_1 = k_2 = 0$.

$\boxed{X_p = \Phi \cdot U}$

$\cancel{X_{geral} = \Phi \cdot (U + C) =} \cancel{\begin{pmatrix} e^{2t}(t^2/2 + t + c_1) + (t+1)e^{2t}(-t + c_2) \\ -e^{2t}(t^2/2 + t + c_1) + (-t-2)e^{2t}(-t + c_2) \end{pmatrix}}$

não é necessário

$X' = \begin{pmatrix} 2 & -2 \\ 2 & 6 \end{pmatrix} X + \begin{pmatrix} 0 \\ e^{4t} \end{pmatrix}$ (2)

a) $X'(t) = \begin{pmatrix} 2 & -2 \\ 2 & 6 \end{pmatrix} X(t)$

autovalores $\det\begin{pmatrix} 2-\lambda & -2 \\ 2 & 6-\lambda \end{pmatrix} = 0 \Rightarrow (2-\lambda)(6-\lambda) + 4 = \lambda^2 - 8\lambda + 16 = 0$

$(\lambda-4)^2 = 0 \quad \lambda = 4$ c/ mult. 2.

autovetores $(A-\lambda I)V = 0$ e $(A-\lambda I)W = V \Rightarrow X_1 = Ve^{\lambda t}$

$X_2 = (Vt + W)e^{\lambda t}$

$\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow 2v_1 + 2v_2 = 0 \Rightarrow v_1 = 1 \quad v_2 = -1 \Rightarrow V = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow 2w_1 + 2w_2 = -1 \Rightarrow w_1 = 1 \quad w_2 = -3/2$

$X_1(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{4t}$ (2)

$X_2(t) = \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} t + \begin{pmatrix} 1 \\ -3/2 \end{pmatrix}\right] e^{4t}$

(2) $X_c(t) = \Phi(t) \cdot C \quad C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ e $\Phi(t) = \begin{pmatrix} e^{4t} & (t+1)e^{4t} \\ -e^{4t} & -(t+\frac{3}{2})e^{4t} \end{pmatrix}$

b) A solução particular $X_p(t) = \Phi(t) \cdot U(t)$

Pelo método de variação de parâmetros

$\det \Phi(t) = -(t+\frac{3}{2})e^{8t} + (t+1)e^{8t} = -\frac{1}{2}e^{8t}$

$U'(t) = \Phi^{-1}(t) \cdot F$

$\Phi^{-1}(t) = \frac{1}{\cancel{\det\Phi}\ -\frac{1}{2}e^{8t}} \begin{pmatrix} -(t+\frac{3}{2})e^{4t} & -(t+1)e^{4t} \\ e^{4t} & e^{4t} \end{pmatrix} = \begin{pmatrix} (2t+3)e^{-4t} & 2(t+1)e^{-4t} \\ -2e^{-4t} & -2e^{-4t} \end{pmatrix}$

$\Phi^{-1} \cdot F = U'(t) = \begin{pmatrix} 2(t+1) \\ -2 \end{pmatrix}$ (2) $\xrightarrow{\text{integrar}} U(t) = \begin{pmatrix} t^2 + 2t + k_1 \\ -2t + k_2 \end{pmatrix}$

podemos assumir $k_1 = k_2 = 0$

(2) $U(t) = \begin{pmatrix} t^2 + 2t \\ -2t \end{pmatrix}$

B2

$$X'(t) = \begin{pmatrix} 1 & -4 \\ 1 & 5 \end{pmatrix} X(t) + \begin{pmatrix} e^{3t} \\ 0 \end{pmatrix}$$

2

a) $X'(t) = \begin{pmatrix} 1 & -4 \\ 1 & 5 \end{pmatrix} X(t)$

autovalores: $\det \begin{pmatrix} 1-\lambda & -4 \\ 1 & 5-\lambda \end{pmatrix} = 0 \Rightarrow (1-\lambda)(5-\lambda)+4 = \lambda^2 - 6\lambda + 9 = 0$

$(\lambda-3)^2 = 0 \quad \lambda = 3$ autovalor repetido.

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autovetores: $(A-\lambda I)V = 0 \quad (A-\lambda I)W = V \Rightarrow \begin{cases} X_1(t) = Ve^{\lambda t} \\ X_2(t) = (Vt+W)e^{\lambda t} \end{cases}$

2

$\begin{pmatrix} -2 & -4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow v_1 + 2v_2 = 0 \Rightarrow v_1 = 2$ e $v_2 = -1 \Rightarrow V = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -2 & -4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Rightarrow w_1 + 2w_2 = -1 \Rightarrow w_1 = -3 \quad w_2 = 1$

$X_1(t) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{3t}$ e $X_2(t) = \left[\begin{pmatrix} 2 \\ -1 \end{pmatrix} t + \begin{pmatrix} -3 \\ 1 \end{pmatrix}\right] e^{3t} \Rightarrow \boxed{\Phi(t) = \begin{pmatrix} 2e^{3t} & (2t-3)e^{3t} \\ -e^{3t} & (-t+1)e^{3t} \end{pmatrix}}$

matriz fundamental

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2 $\quad \boxed{X_c(t) = \Phi(t) \cdot C}$ onde $\boxed{C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}$

(b) A solução particular pelo método de variação de parâmetros

$X_p(t) = \Phi(t)\,u(t)$ e $\boxed{u'(t) = \Phi^{-1}(t).F}$

$\det \Phi(t) = 2(-t+1)e^{6t} + (2t-3)e^{6t} = -e^{6t} \Rightarrow$

$\Rightarrow \Phi^{-1}(t) = \frac{1}{-e^{6t}} \begin{pmatrix} (-t+1)e^{3t} & -(2t-3)e^{3t} \\ e^{3t} & 2e^{3t} \end{pmatrix} = \begin{pmatrix} (t-1)e^{-3t} & (2t-3)e^{-3t} \\ -e^{-3t} & -2e^{-3t} \end{pmatrix}$

$u' = \Phi^{-1}(t) F = \begin{pmatrix} (t-1)e^{-3t} & (2t-3)e^{-3t} \\ -e^{-3t} & -2e^{-3t} \end{pmatrix}\begin{pmatrix} e^{3t} \\ 0 \end{pmatrix} = \begin{pmatrix} t-1 \\ -1 \end{pmatrix}$ 2

$\Rightarrow u(t) = \begin{pmatrix} \int t-1\,dt \\ \int -dt \end{pmatrix} = \begin{pmatrix} \frac{t^2}{2} - t + k_1 \\ -t + k_2 \end{pmatrix}$ Podemos assumir $k_1 = k_2 = 0$

$$\boxed{u(t) = \begin{pmatrix} \frac{t^2}{2} - t \\ -t \end{pmatrix}}$$

2

B3

$$X'(t) = \begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix} X(t) + \begin{pmatrix} e^t \\ 0 \end{pmatrix}$$

2

a) $X'(t) = \begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix} X(t)$

autovalores: $\det \begin{pmatrix} 4-\lambda & -3 \\ 3 & -2-\lambda \end{pmatrix} = 0 \Rightarrow (4-\lambda)(-2-\lambda)+9 = \lambda^2 - 2\lambda + 1 = 0$

$(\lambda-1)^2 = 0$ $\lambda = 1$ autovalor repetido.

autovetores: $(A-\lambda I)V = 0$ e $(A-\lambda I)W = V \Rightarrow \begin{cases} X_1(t) = Ve^{\lambda t} \\ X_2(t) = [Vt + W]e^{\lambda t} \end{cases}$

$\begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow 3v_1 - 3v_2 = 0 \Rightarrow v_1 = v_2 = 1 \Rightarrow V = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2 $\begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow 3w_1 - 3w_2 = 1$ $w_1 = 1$ $w_2 = 2/3$

$X_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$ $X_2(t) = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} t + \begin{pmatrix} 1 \\ 2/3 \end{pmatrix}\right] e^t \Rightarrow \boxed{\Phi(t) = \begin{pmatrix} e^t & (t+1)e^t \\ e^t & (t+\frac{2}{3})e^t \end{pmatrix}}$

matriz fundamental

2 $\boxed{X_c(t) = \Phi(t) \cdot C}$ onde $\boxed{C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}$

(b) A solução particular $X_p(t) = \Phi(t) \cdot u(t)$ 2

Pelo método de variação de parâmetros

$\boxed{u'(t) = \Phi^{-1}(t) F}$ $\det \Phi(t) = (t+\frac{2}{3})e^{2t} - (t+1)e^{2t} = -\frac{1}{3}e^{2t}$

$$\Phi^{-1}(t) = -\frac{3}{e^{2t}} \begin{pmatrix} (t+\frac{2}{3})e^t & -(t+1)e^t \\ -e^t & e^t \end{pmatrix} = \begin{pmatrix} (-3t-2)e^{-t} & (3t+3)e^{-t} \\ 3e^{-t} & -3e^{-t} \end{pmatrix}$$

$$u' = \Phi^{-1}(t) \cdot F = \begin{pmatrix} (-3t-2)e^{-t} & (3t+3)e^{-t} \\ 3e^{-t} & -3e^{-t} \end{pmatrix} \begin{pmatrix} e^t \\ 0 \end{pmatrix} = \begin{pmatrix} -3t-2 \\ 3 \end{pmatrix}$$

$$u(t) = \begin{pmatrix} \int -3t-2\, dt \\ \int 3\, dt \end{pmatrix} = \begin{pmatrix} -\frac{3t^2}{2} - 2t + k_1 \\ 3t + k_2 \end{pmatrix}$$

Podemos assumir que $k_1 = k_2 = 0$

2

$$\boxed{u(t) = \begin{pmatrix} -\frac{3t^2}{2} - 2t \\ 3t \end{pmatrix}} \qquad \boxed{X_p(t) = \Phi(t) \cdot u(t)}$$

$3y'' + 2xy' + (4-x^2)y = 0 \qquad y(0) = -2 \quad y'(0) = 3$

$y'' + P(x)y' + Q(x)y = 0 \Rightarrow P(x) = \frac{2x}{3}$ e $Q(x) = \frac{4-x^2}{3}$ (2)

Como $\lim_{x\to 0} P(x) = 0$ e $\lim_{x\to 0} Q(x) = 4/3$ existem $\Rightarrow x=0$ é ponto ordinário

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \Rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \Rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1)x^{k-2}$

Substituindo:

$$\sum_{k=2}^{\infty} 3c_k k(k-1)x^{k-2} + \sum_{k=1}^{\infty} 2c_k k \underbrace{x^{k-1}x}_{x^k} + \sum_{k=0}^{\infty} 4c_k x^k - \sum_{k=0}^{\infty} c_k \underbrace{x^k x^2}_{x^{k+2}} = 0$$ (2)

deslocamento $k \to k+2$ | deslocamento $k \to k-2$

$$\sum_{k=0}^{\infty} 3c_{k+2}(k+2)(k+1)x^k + \sum_{k=0}^{\infty} 2c_k k x^k + \sum_{k=0}^{\infty} 4c_k x^k - \sum_{k=2}^{\infty} c_{k-2}x^k = 0.$$

retirar os termos em $k=0$ e $k=1$ p/ fora da serie (2)

$$3c_2 2 + \cancel{2c_0 0} + 4c_0 + 3c_3 \cdot 6x + 2c_1x + 4c_1x + \sum_{k=2}^{\infty}\left(3c_{k+2}(k+2)(k+1) + (2k+4)c_k - c_{k-2}\right)x^k = 0$$

$6c_2 + 4c_0 = 0 \Rightarrow c_2 = 8/6 = 4/3$ $\qquad y(0) = c_0 = -2$ (2)

$18c_3 + 6c_1 = 0 \Rightarrow c_3 = -1$ $\qquad y'(0) = c_1 = 3$

fórmula de recorrencia

$$\boxed{c_{k+2} = \frac{c_{k-2} - (2k+4)c_k}{3(k+2)(k+1)}} \quad k \geq 2$$

$k=2 \quad c_4 = \frac{c_0 - 8c_2}{3 \cdot 4 \cdot 3} = -\frac{19}{54}$

$k=3 \quad c_5 = \frac{c_1 - 10c_3}{3 \cdot 5 \cdot 4} = \frac{13}{60}$

$k=4 \quad c_6 = \frac{c_2 - 12c_4}{3 \cdot 6 \cdot 5}$

$\Rightarrow c_6 = \frac{53}{54 \cdot 90} = \frac{53}{4860}$

$\therefore y(x) \approx -2 + 3x + \frac{4}{3}x^2 - x^3 - \frac{19}{54}x^4 + \frac{13}{60}x^5 + \frac{53}{4860}x^6$

(2)

C1 $y'' + 4xy' + (4x^2+2)y = 0 \qquad y(0)=1 \quad y'(0)=1$

$y'' + P(x)y' + Q(x)y = 0 \Rightarrow P(x) = 4x$ e $Q(x) = 4x^2+2$ (2)

Como $\lim_{x\to 0} P(x) = 0$ e $\lim_{x\to 0} Q(x) = 2$ existem $\Rightarrow x_0 = 0$ é ponto ORDINÁRIO

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \Rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \Rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1) x^{k-2}$

Substituindo:

$$\sum_{k=2}^{\infty} c_k k(k-1)x^{k-2} + \sum_{k=1}^{\infty} 4c_k k \underbrace{x^{k-1} x}_{x^k} + \sum_{k=0}^{\infty} 4c_k \underbrace{x^k x^2}_{x^{k+2}} + \sum_{k=0}^{\infty} 2c_k x^k = 0$$ (2)

deslocamento $k \to k+2$ | deslocamento $k \to k-2$ (2)

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)x^k + \sum_{k=0}^{\infty} 4c_k k x^k + \sum_{k=2}^{\infty} 4c_{k-2} x^k + \sum_{k=0}^{\infty} 2c_k x^k = 0$$

$\Rightarrow$ precisamos começar todas as séries em $k=2$

$\Rightarrow$ retirar termos $k=0$ e $k=1$

$$\underbrace{[2c_2 + 2c_0]}_{k=0;\ =0} + \underbrace{[6c_3 + 4c_1 + 2c_1]}_{k=1;\ =0} x + \sum_{k=2}^{\infty} \underbrace{[c_{k+2}(k+2)(k+1) + (4k+2)c_k + 4c_{k-2}]}_{=0} x^k = 0$$

Princípio da Identidade $\Rightarrow$

$y(0) = \boxed{c_0 = 1} \qquad y'(0) = \boxed{c_1 = 1}$

$c_2 = -c_0 \quad \boxed{c_2 = -1} \qquad c_3 = -c_1 \Rightarrow \boxed{c_3 = -1}$ (2)

$$\boxed{c_{k+2} = \frac{-(4k+2)c_k - 4c_{k-2}}{(k+2)(k+1)}} \qquad k \geq 2$$

$k=2 \quad c_4 = \dfrac{-(10)c_2 - 4c_0}{4 \cdot 3} = \dfrac{6}{12} = \dfrac{1}{2} \qquad \boxed{c_4 = \frac{1}{2}}$

$k=3 \quad c_5 = \dfrac{-(14)c_3 - 4c_1}{5 \cdot 4} = \dfrac{10}{20} \Rightarrow \boxed{c_5 = \frac{1}{2}}$

$k=4 \quad c_6 = \dfrac{-(18)c_4 - 4c_2}{6 \cdot 5} = \dfrac{-5}{6 \cdot 5} \Rightarrow \boxed{c_6 = -\frac{1}{6}}$

$k=5 \quad c_7 = \dfrac{(22)c_5 - 4c_3}{7 \cdot 6} = -\dfrac{7}{42} = -\dfrac{1}{6} \Rightarrow \boxed{c_7 = -\frac{1}{6}}$ (2)

$$y(x) \approx 1 + x - x^2 - x^3 + \frac{x^4}{2} + \frac{x^5}{2} - \frac{x^6}{6} - \frac{x^7}{6}$$

C2

$$(1-2x^3)y'' + 6x^2y' + 24xy = 0 \qquad y(0)=1 \quad y'(0)=1$$

(2)

$$y'' + P(x)y' + Q(x)y = 0 \Rightarrow P(x) = \frac{6x^2}{1-2x^3} \text{ e } Q(x) = \frac{24x}{1-2x^3}$$

$$\lim_{x\to 0} P(x) = 0 \text{ e } \lim_{x\to 0} Q(x) = 0 \Rightarrow \text{Como ambos os limites existem}$$

$$\Rightarrow x_0 = 0 \text{ é ponto ordinário}$$

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1)x^{k-2}$ (2)

Substituindo:

$$\sum_{k=2}^{\infty} c_k k(k-1)x^{k-2} - \sum_{k=2}^{\infty} 2c_k k(k-1)\underbrace{x^{k-2}x^3}_{\to x^{k+1}} + \sum_{k=1}^{\infty} 6c_k k \underbrace{x^{k-1}x^2}_{\to x^{k+1}} + \sum_{k=0}^{\infty} 24 c_k \underbrace{x^k x}_{\to x^{k+1}} = 0$$

deslocamentos

$k \to k+2$ $\quad k \to k-1$ $\quad k \to k-1$ $\quad k \to k-1$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)x^k - \sum_{k=3}^{\infty} 2c_{k-1}(k-1)(k-2) + \sum_{k=2}^{\infty} 6c_{k-1}(k-1)x^k + \sum_{k=1}^{\infty} 24c_{k-1}x = 0$$

(2)

$k=1$ pois $2c_{k-1}(k-1)(k-2)=0$ para $k=1$ e $k=2$

$k=1$ pois $6c_{k-1}(k-1)=0$ para $k=1$

$k=0$

$$\underbrace{2c_2}_{=0} + \sum_{k=1}^{\infty} \underbrace{\left[c_{k+2}(k+2)(k+1) + \left(-2(k-1)(k-2) + 6(k-1) + 24\right)c_{k-1}\right]}_{=0} x^k = 0$$

pelo Princípio da Identidade

$$\boxed{c_2 = 0} \qquad c_{k+2} = \frac{2\left[\overbrace{(k-1)(k-2) - 3(k-1) - 12}^{k^2-6k-7}\right]c_{k-1}}{(k+2)(k+1)} \qquad k \geq 1$$

$$\boxed{c_{k+2} = \frac{2(k-7)\cancel{(k+1)}\,c_{k-1}}{(k+2)\cancel{(k+1)}}} \qquad k \geq 1 \qquad (2)$$

$k^2 - 6k - 7 = 0$

$$\frac{6 \pm \sqrt{36+28}}{2} = \frac{6 \pm 8}{2} \begin{cases} 7 \\ -1 \end{cases}$$

$k=1$ $\quad c_3 = \frac{2(-6)}{3}c_0$ $\qquad k=2$ $\quad c_4 = \frac{2(-5)}{4}c_1$ $\qquad k=3$ $\quad c_5 = *\cdot \underset{0}{c_2} = 0$

$k=4$ $\quad c_6 = \frac{2(-3)}{6}c_3 = \frac{2(-3)}{6}\frac{(2)(-6)}{3}c_0$

$k=5$ $\quad c_7 = \frac{2(-2)}{7}c_4 = \frac{2(-2)}{7}\cdot\frac{2(-5)}{4}c_1$ $\qquad$ (2)

$k=6$ $\quad c_8 = \frac{2(-1)}{8}c_5 = 0$

$k=7$ $\quad c_9 = \frac{0}{9}c_8 = 0$

$k=8$ $\quad c_{10} = \frac{2(1)}{10}c_7 = \frac{2.(1)}{10}\frac{2(-2)\,2(-5)}{7\,.\,4}c_1$

$y(0) = c_0 = 1$
$y'(0) = c_1 = 1$
$c_2 = 0$
$c_3 = -4$
$c_4 = -5/2$ $\quad c_5 = 0$
$c_6 = 4$
$c_7 = 10/7$
$c_8 = 0$
$c_9 = 0$ $\quad c_{10} = 2/7$

$$y(x) \approx 1 + x - 4x^3 - \frac{5}{2}x^4 + 4x^6 + \frac{10}{7}x^7 + \frac{2}{7}x^{10}$$

C3

$y'' + x^6 y' + 7x^5 y = 0 \qquad y(0) = 2 \quad y'(0) = 3$ (2)

$y'' + P(x)y' + Q(x)y = 0 \Rightarrow P(x) = x^6$ e $Q(x) = 7x^5$. Como

$\lim_{x\to 0} P(x) = 0$ e $\lim_{x\to 0} Q(x) = 0$ existem $\Rightarrow x = 0$ é ponto ordinár

Suponha $y(x) = \sum_{k=0}^{\infty} c_k x^k \rightarrow y'(x) = \sum_{k=1}^{\infty} c_k k x^{k-1} \rightarrow y''(x) = \sum_{k=2}^{\infty} c_k k(k-1) x^{k-2}$

Substituindo (2)

$$\sum_{k=2}^{\infty} c_k k(k-1) x^{k-2} + \sum_{k=1}^{\infty} c_k k \underbrace{x^{k-1} x^6}_{x^{k+5}} + \sum_{k=0}^{\infty} 7 c_k \underbrace{x^k x^5}_{x^{k+5}} = 0$$

deslocamentos

$k \to k+2 \qquad k \to k-5 \qquad k \to k-5$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)x^k + \sum_{k=6}^{\infty} c_{k-5}(k-5)x^k + \sum_{k=5}^{\infty} 7c_{k-5} x^k = 0$$ (2)

$\rightarrow k=5$ pois $c_{k-5}(k-5) = 0$ em $k=5$

Preciso retirar os termos $k = 0, 1, 2, 3, 4$

$$\underbrace{2c_2}_{=0} + \underbrace{6c_3}_{=0} x + \underbrace{12c_4}_{=0} x^2 + \underbrace{20c_5}_{=0} x^3 + \underbrace{30c_6}_{=0} x^4 + \sum_{k=5}^{\infty} \underbrace{\left[c_{k+2}(k+2)(k+1) + (k+2)c_{k-5}\right]}_{=0} x^k = 0$$

pelo Princípio da Identidade

$\Rightarrow c_2 = c_3 = c_4 = c_5 = c_6 = 0$

rel. de recorrência $\quad c_{k+2} = \dfrac{-(\cancel{k+2})c_{k-5}}{(\cancel{k+2})(k+1)} = \dfrac{-c_{k-5}}{k+1} \quad k \geq 5$ (2)

$k=5 \quad c_7 = -\dfrac{c_0}{6} \qquad k=6 \quad c_8 = -\dfrac{c_1}{7}$

$k = 7, 8, 9, 10, 11 \quad c_9 = * c_2 = 0; \; c_{10} = * c_3 = 0; \; c_{11} = * c_4 = 0; \; c_{12} = * c_5 = 0$

$c_{13} = * c_6 = 0$

$k = 12 \qquad c_{14} = -\dfrac{c_7}{13} = -\dfrac{-c_0}{13.6}$

$k = 13 \qquad c_{15} = -\dfrac{c_8}{14} = -\dfrac{-c_1}{14.7}$

$y(0) = c_0 = 2$

$y'(0) = c_1 = 3$ (2)

$$y(x) \approx 2 + 3x - \frac{2}{6}x^7 - \frac{3}{7}x^8 + \frac{1}{39}x^{14} + \frac{3}{98}x^{15}$$

DO

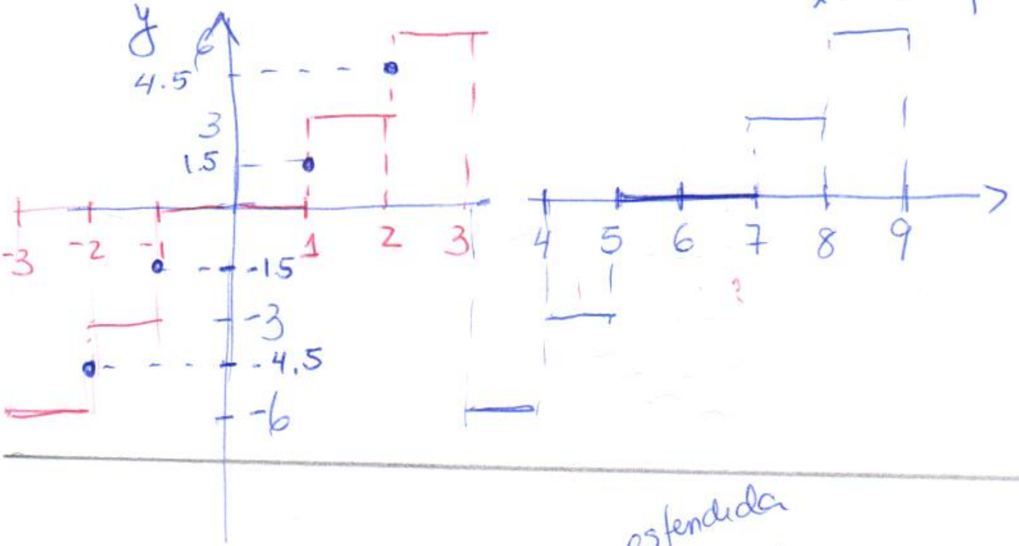
$$f(x) = \begin{cases} 0 & 0<x<1 \\ 3 & 1<x<2 \\ 6 & 2<x<3 \end{cases}$$

$x=1 \quad f(x)=1.5$
$x=2 \quad f(x)=4.5$
$x=-1 \quad f(x)=-1.5$
$x=-2 \quad f(x)=-4.5$

$x=3 \quad f(x)=0$
$x=-3 \quad f(x)=0$

2

a)



2

$2L = 6$

$\boxed{L=3}$

b) Como a função $\overset{\text{estendida}}{}$ é ímpar $\Rightarrow$ 2

$a_0 = 0 \qquad a_n = 0$

$$b_n = \frac{1}{3}\int_{-3}^{3} \underbrace{f_{ext.\,impar}}_{imp.} \underbrace{sen\frac{n\pi x}{3}}_{imp}\,dx = \frac{2}{3}\int_0^3 f(x)\, sen\frac{n\pi x}{3}\,dx$$

par

$$\frac{2}{3}\left[\int_0^1 0\,dx + \int_1^2 3\, sen\frac{n\pi x}{3}dx + \int_2^3 6\, sen\frac{n\pi x}{3}\,dx\right] =$$ 2

$$\frac{2}{3}\left[-\frac{3.3}{n\pi}\cos\frac{n\pi x}{3}\Big|_1^2 - \frac{6.3}{n\pi}\cos\frac{n\pi x}{3}\Big|_2^3\right] =$$

$$-\frac{6}{n\pi}\left(\cos\frac{2n\pi}{3} - \cos\frac{n\pi}{3}\right) - \frac{12}{n\pi}\left(\cos n\pi - \cos\frac{2n\pi}{3}\right)$$

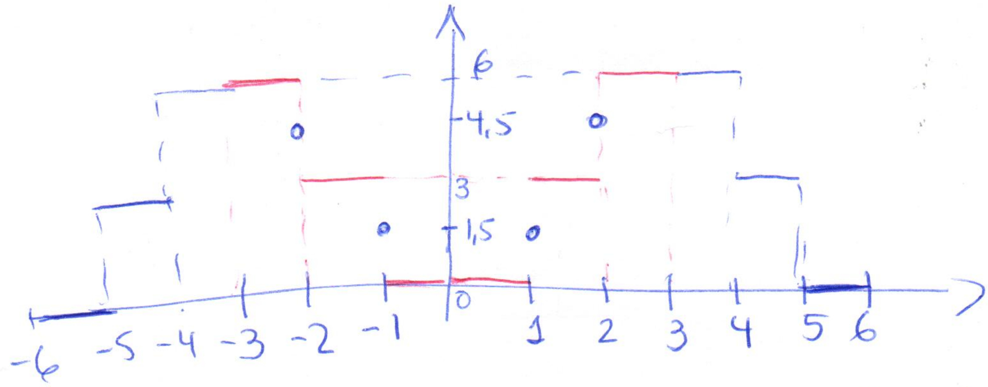
$$\frac{6}{n\pi}\cos\frac{2n\pi}{3} + \frac{6}{n\pi}\cos\frac{n\pi}{3} - \frac{12}{n\pi}(-1)^n = b_n$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{6}{n\pi}\cos\frac{2n\pi}{3} + \frac{6}{n\pi}\cos\frac{n\pi}{3} - \frac{(-1)^n 12}{n\pi}\right) sen\frac{n\pi x}{3}$$

2

D1

a)



2

$2L = 6$
$L = 3$

$f(0) = 0$
$f(1) = 1{,}5 = f(-1)$
$f(2) = 4{,}5 = f(-2)$
$f(3) = 6 = f(-3)$

2

b) Como a função estendida é par

$\Rightarrow b_n = 0$

2

$$a_0 = \frac{1}{3}\int_{-3}^{3} f_{ext\ par}(x)\,dx = \frac{2}{3}\left[\int_0^1 0\,dx + \int_1^2 3\,dx + \int_2^3 6\,dx\right]$$

$$= \frac{2}{3}\left[3x\Big|_1^2 + 6x\Big|_2^3\right] = \frac{2}{3}\left[(\cancel{6}-3) + (\cancel{18}-12)\right] = 6$$

2

$$a_n = \frac{1}{3}\int_{-3}^{3} \underbrace{f_{ext\ par}}_{par}\ \underbrace{\cos\frac{n\pi x}{3}}_{par}\,dx = \frac{2}{3}\int_0^3 f_{ext\ par}\cos\frac{n\pi x}{3}\,dx =$$

par

$$= \frac{2}{3}\left[\int_0^1 0 + \int_1^2 3\cos\frac{n\pi x}{3}\,dx + \int_2^3 6\cos\frac{n\pi x}{3}\right] = \frac{2}{3}\left[3.\frac{3}{n\pi}\,\text{sen}\frac{n\pi x}{3}\Big|_1^2 + 6.\frac{3}{n\pi}\,\text{sen}\frac{n\pi x}{3}\Big|_2^3\right] = \frac{6}{n\pi}\left(\text{sen}\frac{2n\pi}{3} - \text{sen}\frac{n\pi}{3}\right) + \frac{12}{n\pi}\left(\underbrace{\text{sen}\,n\pi}_{0} - \text{sen}\frac{2n\pi}{3}\right)$$

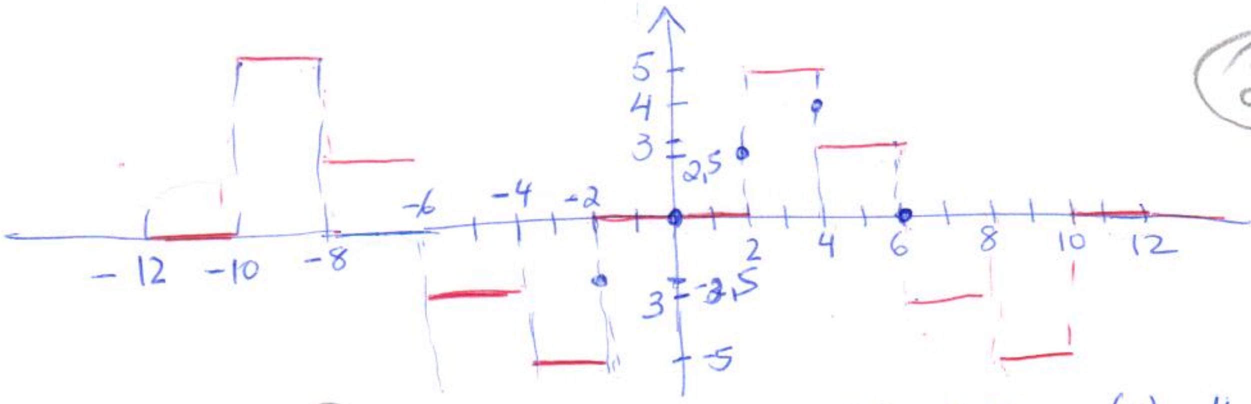
$$= \boxed{-\frac{6}{n\pi}\,\text{sen}\frac{2n\pi}{3} - \frac{6}{n\pi}\,\text{sen}\frac{n\pi}{3} = a_n}$$

$$f(x) = \underbrace{3}_{\frac{a_0}{2}} + \sum_{n=1}^{\infty}\left(-\frac{6}{n\pi}\right)\left[\text{sen}\frac{2n\pi}{3} + \text{sen}\frac{n\pi}{3}\right]\cos\frac{n\pi x}{3}$$

2

V2

$$f(x) = \begin{cases} 0 & 0 < x < 2 \\ 5 & 2 < x < 4 \\ 3 & 4 < x < 6 \end{cases}$$



2

2 $\rightarrow$ $f(0)=0$ $\quad f(2) = 2,5 \quad f(4) = 4 \quad f(6)=0$

$f(-2) = -2,5 \quad f(-4) = -4 \quad f(-6) = 0$

Como a função estendida é ímpar:

$\Rightarrow a_0 = 0$ e $a_n = 0$ $\qquad P = 12 \Rightarrow L = 6$ 2

2 $b_n = \frac{1}{6}\int_{-6}^{6} \underbrace{f_{ext.imp.}}_{ímpar} \underbrace{\operatorname{sen}\frac{n\pi x}{6}}_{ímpar} dx = \frac{2}{6}\int_0^6 f_{ext.imp.} \operatorname{sen}\frac{n\pi x}{6} dx$

par

$= \frac{1}{3}\left[\int_0^2 0\,dx + \int_2^4 3 \operatorname{sen}\frac{n\pi x}{6} + \int_4^6 5 \operatorname{sen}\frac{n\pi x}{6} dx\right] = \frac{1}{3}\left[-3 \cdot \frac{6}{n\pi} \cos\frac{n\pi x}{6}\Big|_2^4 - 5\frac{6}{n\pi} \cos\frac{n\pi x}{6}\Big|_4^6\right]$

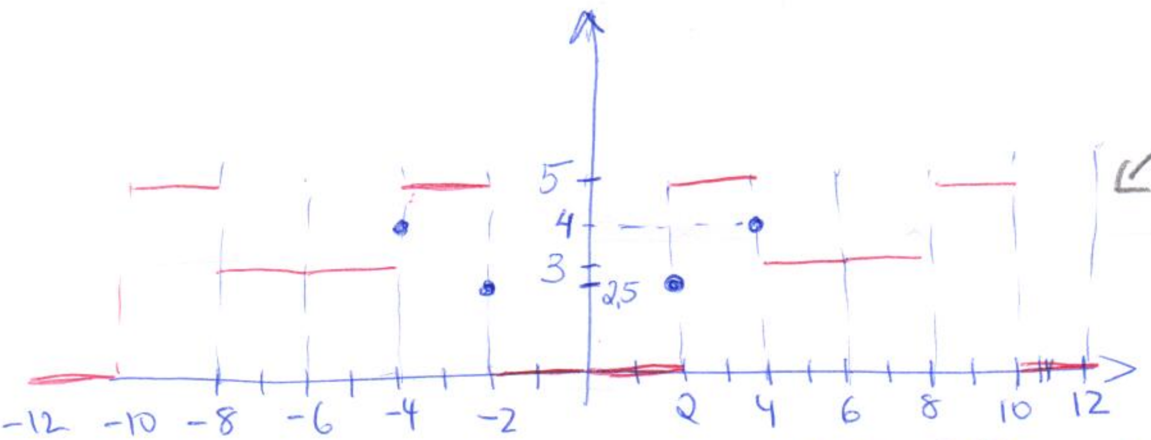
$= -\frac{6}{n\pi}\left(\cos\frac{4n\pi}{6} - \cos\frac{2n\pi}{6}\right) - \frac{10}{n\pi}\left(\cos n\pi - \cos\frac{4n\pi}{6}\right) =$

$= \frac{4}{n\pi}\cos\frac{4n\pi}{6} + \frac{6}{n\pi}\cos\frac{2n\pi}{6} - \frac{10}{n\pi}(-1)^n = \boxed{\frac{1}{n\pi}\left(4\cos\frac{2n\pi}{3} + 6\cos\frac{n\pi}{3} - 10(-1)^n\right) = b_n}$

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n\pi}\left(4\cos\frac{2n\pi}{3} + 6\cos\frac{n\pi}{3} - (-1)^n \cdot 10\right) \operatorname{sen}\frac{n\pi x}{6}$$

2

P3



2

$f(0)=0$ $\quad f(2)=f(-2)=2{,}5$ $\quad f(6)=f(-6)=3$

$P=2L=12 \Rightarrow L=6$ $\quad f(4)=f(-4)=4$ 2

Como a função estendida é par $\Rightarrow b_n=0$ 2

$$a_0=\frac{1}{6}\int_{-6}^{6} f_{ext\ par}\,dx=\frac{2}{6}\int_{0}^{6} f_{ext\ par}\,dx=\frac{1}{3}\left[\int_0^2 0\,dx+\int_2^4 5\,dx+\int_4^6 3\,dx\right]$$

$$=\frac{1}{3}\left[5x\Big|_2^4+3x\Big|_4^6\right]=\frac{1}{3}\left[(\underbrace{20-10}_{10})+(\underbrace{18-12}_{6})\right]=\frac{16}{3}$$ 2

$$a_n=\frac{1}{6}\int_{-6}^{6} \underbrace{f_{ext\ par}}_{par}\ \underbrace{\cos\frac{n\pi x}{6}}_{par}\,dx=\frac{2}{6}\int_0^6 f_{ext\ par}\cos\frac{n\pi x}{6}\,dx=$$

par

$$=\frac{1}{3}\left[\int_0^2 0\,dx+\int_2^4 5\cos\frac{n\pi x}{6}\,dx+\int_4^6 3\cos\frac{n\pi x}{6}\,dx\right]=\frac{1}{3}\left[\frac{5.6}{n\pi}\,\text{sen}\,\frac{n\pi x}{6}\Big|_2^4+\frac{3.6}{n\pi}\,\text{sen}\,\frac{n\pi x}{6}\Big|_4^6\right]$$

$$=\frac{10}{n\pi}\left(\text{sen}\,\frac{4n\pi}{6}-\text{sen}\,\frac{2n\pi}{6}\right)+\frac{6}{n\pi}\left(\underbrace{\text{sen}\,n\pi}_{0}-\text{sen}\,\frac{4n\pi}{6}\right)=\frac{4}{n\pi}\,\text{sen}\,\frac{4n\pi}{6}-\frac{10}{n\pi}\,\text{sen}\,\frac{2n\pi}{6}$$

$$=\frac{1}{n\pi}\left(4\,\text{sen}\,\frac{2n\pi}{3}-10\,\text{sen}\,\frac{n\pi}{3}\right)=a_n$$

$$f(x)=\frac{16}{6}+\sum_{n=1}^{\infty}\frac{1}{n\pi}\left(4\,\text{sen}\,\frac{2n\pi}{3}-10\,\text{sen}\,\frac{n\pi}{3}\right)\cos\frac{n\pi x}{6}$$ 2

$\frac{a_0}{2}$